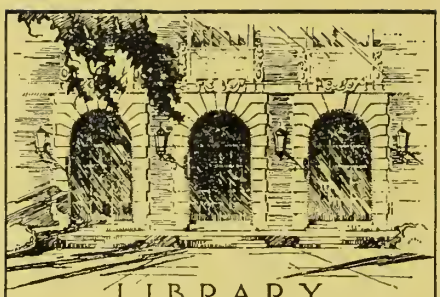

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REPORT NO. 143

PROPERTIES OF INTERCOUPLED TRANSMISSION LINES TERMINATED BY NEGATIVE RESISTANCE ELEMENTS WITH APPLICATIONS TO TUNNEL DIODE PULSE CIRCUITS

by

Henry Guckel

June 28, 1963

(This work is being submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy in Electrical Engineering, June, 1963, and was supported in part by the Office of Naval Research under contract Nonr-1834(15).)

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ABSTRACT

Transmission of nanosecond and subnanosecond pulse-coded information between successive processors can be accomplished only if the physical interconnection takes the form of a transmission line terminated in its characteristic impedance, i.e., if reflectionless, distortion-free transmission between processors occurs. This fact is used as the basis for the investigation of intercoupled transmission lines. Differential matrix equations are developed from Maxwell's equations applied to multiconductor systems. The fact that wideband application is desired is used to restrict the general solutions to expressions which permit the propagation of a single TEM mode only. It is shown that lossy intercoupled transmission line systems can be constructed with the property of distortion-free, single-mode propagation (generalized Heaviside systems). The specialization of this type of network to two intercoupled lines, which in turn are restricted by unidirectional propagation requirements, leads to the double-pair transformer. The suppression of the two reflected waves is possible if the termination impedances are functionally related to each other. These relationships are derived and their physical implications are interpreted to demand that at least one of the termination impedances is an active impedance. If the terminations are resistances of both sign reflectionless, distortionless transmission can be obtained for all frequencies. Properties of such a system include the possibility of voltage and current gain as well as a resistive input impedance. Stability considerations are introduced by establishing the stability restrictions for a simple active network in terms of which the general active circuit may be represented. The Nyquist criterion as well as complex frequency root location constraints are used in the analysis. The linear treatment is extended to those nonlinear active elements which can be described by piecewise linear approximations. Interconnection rules are established which relate

system stability and element stability. These arguments are used as the basis for the stability analysis of the double-pair transformer. The effects of a capacitance in parallel with the negative resistance are studied, and it is concluded that intercoupled systems terminated by a combination of tunnel diodes and positive resistances do possess various forms of gain and are stable.

The theoretical arguments have been based on the existence of a constant negative resistance. Its realization from tunnel diodes is investigated. The magnitudes of tolerances to be expected are discussed and their effects on the ideal system are studied. Numerical examples based on computer calculations are given. The physical realization of intercoupled transmission lines is examined. A T-line made from two copper-clad dielectric sheets is discussed. Design data obtained from computer calculations for broadside-coupled strip transmission lines are given. It is concluded the the mathematical model may be approximated well enough in practical realizations to retain its desired characteristics. These are: (1) reflectionless transmission and, therefore, constant, resistive input impedance, (2) possible voltage and current gains which are independent of frequency for frequency bands as large as 25-2,500 megacycles. These statements are supported by experimental data.

INTRODUCTION

A statement concerning the completeness of a set of logical circuits includes as a necessary condition the fanin and fanout properties of the circuits. If the representation of logic signals is taken as two voltage bands and all circuits of the set have the same input properties, power gain occurs within the logic elements if the fanout exceeds the fanin and output levels are in the specified voltage bands. Furthermore, since an input signal which is near the lower band edge must result in an output signal which is still within the specified band, a voltage gain of at least unity must occur. A favorable fanin-fanout relationship requires in this case both voltage and current gain.

If the active element involved in these circuits is a two-terminal element, the requirement that a change at the output terminals does not affect the input states, i.e., that the element is directive, causes additional restrictions. The method which is used to provide directivity must be chosen in such a way that it imposes minimum restrictions on the actual logic path, i.e., it does not influence the path which it must control. If directivity controls are inserted as separate mechanisms they should be as simple as possible and their incorporation as part of the logic circuit offers the most advantages. Since the active elements involved in these circuits must impart quantization[8] to the signals their nonlinearity is implied. The fact that they are nonlinear allows the use of linear amplifiers with high forward-to-backward transmission ratios as one form of directivity imparting elements. Restoration is accomplished simultaneously.

An all-tunnel-diode logic system, which utilizes voltage representation, would obviously be brought one step closer to realization if an amplifier could be realized which has both voltage and current gain. Furthermore, since dc-coupling would result in bias and signal loading simultaneously, an ac-coupling

method is preferred. Dc-restoration could be accomplished by utilizing the inherent memory capability of the N type nonlinearity of the diode. In order to utilize fully the switching capability of the active element the coupling circuit must be a low pass filter with a pass band which extends beyond the cutoff frequency of the diode. Design considerations for such a device involve minimum free space wave lengths in the decimeter region. This means that lumped circuit concepts are no longer applicable. The fact that signals must be transferred from logic block to logic block enforces this point of view. However, the transmission of pulse shapes between two physically separated nonlinear elements by means of a transmission line is in itself a difficult problem. Reflectionless transmission can only occur if the line is properly terminated; therefore, the nonlinearities of the active elements must be eliminated from the terminations. This nearly always requires some form of damping network with signal dissipation characteristics. An alternative to this solution may be obtained. If the actual performance of some given logical statement is separated from restoration, as for instance in the case of diode-emitter follower logic, the restoring amplifier in the form of an active, ac-coupled, linear distributed amplifier could be used as the physical interconnection. Its realization would attempt to solve the problems of restoration, directivity and interconnection. Since the construction of suitable NOT, OR and AND circuits has already been solved by many investigators, a complete tunnel diode system would become possible. Attempts to realize such an amplifying structure prompted the following more general analysis.

1. INTERCOUPLED TRANSMISSION LINES

1.1 Parallel System of Conductors

It is assumed that a system of perfect, infinitely long conductors exists and is located in a homogeneous, isotropic, lossless medium of constant permeability μ and permittivity ϵ. The interconductor spacing is chosen in such a way that only the TEM mode propagates. Reference coordinates are indicated in Fig. 1.1.

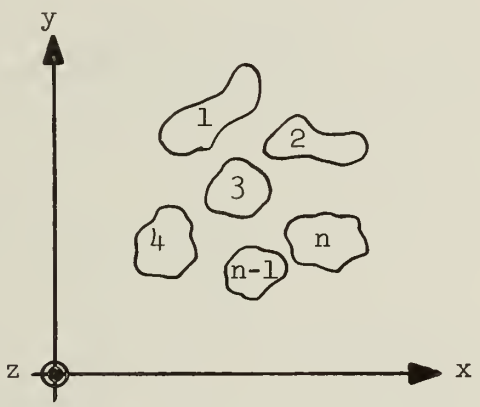

FIGURE 1.1. REFERENCE COORDINATES FOR THE CONDUCTOR SYSTEM

Application of Maxwell's equations yields for the space between the conductors:

$$\bar{\nabla} \times \bar{E} = - \frac{\partial \bar{B}}{\partial t} \tag{1.1}$$

$$\bar{\nabla} \times \bar{H} = \epsilon \frac{\partial \bar{E}}{\partial t} \tag{1.2}$$

$$E_z = H_z = 0 \tag{1.3}$$

The Helmholtz equations are derivable from the preceding differential equations.

$$\bar{\nabla}^2\bar{F} - \mu\epsilon \frac{\partial^2 \bar{F}}{\partial t^2} = 0 \qquad (1.4)$$

$$\bar{F} = \bar{E},\ \bar{H}$$

Initial conditions are assumed to satisfy equation (1.5).

$$\mathcal{L}\left(\frac{\partial^2 \bar{E}(\bar{r},t)}{\partial t^2}\right) = s^2 \bar{E}(\bar{r},s) \qquad \text{(Laplace transform)} \qquad (1.5)$$

The TEM assumption (Eq. 1.3) is used to obtain the one-dimensional Helmholtz equation.

$$\frac{\partial^2 \bar{F}}{\partial z^2} - \mu\epsilon s^2 \bar{F} = 0 \qquad (1.6)$$

$$\bar{F} = \bar{E},\ \bar{H}$$

The curl of both fields has a zero component in the z-direction. Since

$$\bar{\nabla} \times (\bar{\nabla} f) = 0$$

$$f = \text{scalar function}$$

the fields in the transverse plane are derivable from respective potentials.

$$\bar{E} = -\bar{\nabla}\phi \qquad (1.7)$$

$$\bar{H} = \bar{\nabla} \times \bar{A} \qquad (1.8)$$

$$\bar{A} = \hat{z}A = \hat{z}\psi$$

The potential ϕ is the electric potential. ψ denotes the scalar stream function and is equal to the magnitude of the magnetic vector potential $\bar{A}$.

The tangential component of the electric field and the normal component of the magnetic field must vanish on the perfect conductor surfaces. Therefore, the conductors are equipotentials of both ϕ and ψ. Their dependence can be established by considering their definitions (Eqs. (1.7, 1.8)) and their functional relationships (Maxwell's equations).

$$\psi = -\frac{1}{\mu s}\frac{\partial \phi}{\partial z} + \text{constant} \tag{1.9}$$

$$\phi = -\frac{1}{\epsilon s}\frac{\partial \psi}{\partial z} + \text{constant} \tag{1.10}$$

Therefore, ϕ and ψ can differ only by a constant which may depend on z.[11]

1.2 Transverse Capacities

The static field equations hold for any transverse plane. This is a direct consequence of the TEM assumption. The time-independent conductor potentials (v) are directly proportional to the conductor charges (q).

$$[v] = [p][q] \tag{1.11}$$

[p] denotes the matrix formed from Maxwell's potential coefficients. Its inverse [c] is the capacitance coefficient matrix. The charged conductor system is defined as a closed system if it has the property that all lines of flux emanate and terminate on members of the system only. By the use of Gauss' flux theorem such a system is shown to have a zero net charge.

$$\sum_{i=1}^{n} q_i = 0 \tag{1.12}$$

The aggregate of n-charged conductors is therefore described by n-1 linearly-independent equations. [p] is of order (n - 1) x (n - 1). This is equivalent

to the fact that the potential may only be specified within an arbitrary constant, i.e., the zero of potential may be selected.

The capacitance coefficients may be transformed to the direct capacitances.[10]

$$c_{ii} = \sum_{\substack{j=1 \\ j\neq i}}^{n-1} \bar{C}_{ij} \tag{1.13}$$

$$c_{ij} = -\bar{C}_{ij}$$

The closed system is describable by $\frac{n(n-1)}{2}$ distinct capacitors. Furthermore, since the existence of ϕ and ψ is based on the assumption of zero total charge, their existence is assured if the system is closed.

1.3 Transformation to the Circuit Equations

The normalization of the charge q with respect to the line length ℓ results in the line charge λ and capacitance definitions on a per-unit-length basis. Equation (1.11) may be rewritten as

$$[\lambda] = [c][v] \tag{1.14}$$

This equation and the continuity equation (1.15) result in equation (1.16).

$$\frac{\partial}{\partial z}[I] + s[\lambda] = 0 \tag{1.15}$$

$$[v] = \frac{1}{s}[p]\frac{\partial}{\partial z}[I] \tag{1.16}$$

But V_i is given by

$$V_i = \int_{C_0}^{C_i} \overline{\nabla\phi} \cdot \overline{d\ell} = \phi(C_i) - \phi(C_j)$$

$$= \phi(C_i) \equiv \phi_i$$

Here C_i denotes the surface of the ith conductor. The conductor potential of the jth conductor is selected as the zero reference. Equation (1.10) may then be written as

$$[V_i] = - \frac{1}{\epsilon s} \frac{\partial}{\partial z} [\psi_i] \tag{1.17}$$

Comparison with the preceding equation yields as a possible value for $[\psi_i]$:

$$[\psi_i] = \epsilon[p][I] \tag{1.18}$$

However, Eq. (1.9) may also be modified.

$$[\psi_i] = - \frac{1}{\mu s} \frac{\partial}{\partial z} [V] \tag{1.19}$$

Elimination of the stream functions yields the second differential equation.

$$\frac{\partial[V]}{\partial z} = - s\mu\epsilon[p][I] \tag{1.20}$$

The equivalent inductances and capacitances are most conveniently defined as

$$[L] \equiv \mu\epsilon[p]$$

$$\left[\frac{1}{C}\right] \equiv [p] \tag{1.21}$$

The proposed system is therefore described by the two differential equations:

$$[V] = -\frac{1}{s}\left[\frac{1}{C}\right]\frac{\partial}{\partial z}[I]$$

$$\frac{\partial}{\partial z}[V] = -s[L][I] \tag{1.22}$$

The reason for this particular equivalent reactance definition becomes apparent when the second-order differential equation is considered. Since

$$L_{ij}C_{ij} = \mu\epsilon \qquad i, j = 1, 2, \ldots, n-1 \tag{1.23}$$

An equivalent description is given by equation (1.24).

$$\frac{\partial^2}{\partial z^2}[V] = s^2\mu\epsilon[\delta_{ij}][V] \tag{1.24}$$

Therefore, the second-order differential equation is decoupled.

The physical meaning of the three defined capacitances is clarified by considering Example 1.1.

Example 1.1. Three Conductor System

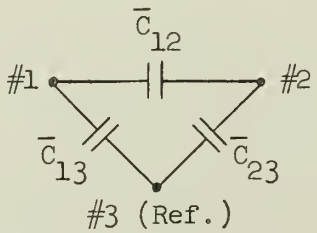

FIGURE 1.2. THREE CONDUCTOR SYSTEM

$$[c] = \begin{bmatrix} \bar{C}_{13}+\bar{C}_{12} & +\bar{C}_{12} \\ +\bar{C}_{12} & \bar{C}_{12}+\bar{C}_{23} \end{bmatrix} = \mu\epsilon[L]^{-1}$$

$$[p] = \frac{1}{(\bar{C}_{13} + \bar{C}_{12})(\bar{C}_{23} + \bar{C}_{12}) - \bar{C}_{12}^2} \begin{bmatrix} \bar{C}_{12}+\bar{C}_{23} & \bar{C}_{12} \\ +\bar{C}_{12} & \bar{C}_{13}+\bar{C}_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{\bar{C}} \end{bmatrix}$$

If the system is symmetric, that is if $\bar{C}_{13} = \bar{C}_{23} = \bar{C}$, the equivalent capacitors are simply:

$$[C] = \bar{C}(\bar{C} + 2\bar{C}_{12}) \begin{bmatrix} \frac{1}{\bar{C}+\bar{C}_{12}} & \frac{1}{\bar{C}_{12}} \\ \frac{1}{\bar{C}_{12}} & \frac{1}{\bar{C}+\bar{C}_{12}} \end{bmatrix}$$

The input impedance of the circuit of Fig. 1.2 between terminals #1 and #3 or #2 and #3 is given by:

$$Z_{ii} = s \frac{\bar{C} + \bar{C}_{12}}{\bar{C}(\bar{C} + 2\bar{C}_{12})} = s \frac{1}{C_{ii}} \qquad i = 1, 2$$

The equivalent capacitor C_{ii} represents the input capacitance between conductor i and the reference.

The transfer impedance from #1 to #2 or #2 to #1 is given by:

$$Z_{ij} = Z_{ii} \frac{\bar{C}_{12}}{\bar{C} + \bar{C}_{12}} = s \frac{\bar{C}_{12}}{\bar{C}(\bar{C} + 2\bar{C}_{12})} = s \frac{1}{C_{ij}} \qquad \begin{matrix} i = 1 \\ j = 2 \end{matrix} \quad \text{or} \quad \begin{matrix} i = 2 \\ j = 1 \end{matrix}$$

Hence, the elements of $\left[\frac{1}{C}\right]$ are related to the impedance matrix by Eq. (1.22).

$$\left[\frac{1}{C}\right] = \frac{1}{s}[z] \tag{1.22}$$

The derivations of the preceding section have been based on the assumptions of perfect conductors in a lossless medium. If lossy conductors are considered the derivation of circuit equations from field equations becomes more difficult.(7) The potential solutions of Maxwell's equations can no longer be based on the fact that the conductor surfaces are equipotentials of the stream function and the electric potential. However, it may be assumed that the form of the two differential equations is preserved. The series impedance and shunt admittance are in this case defined by the differential equations:

$$\frac{\partial[V]}{\partial z} = -[Z][I] \tag{1.23}$$

$$\frac{\partial[I]}{\partial z} = -[Y][V] \tag{1.24}$$

The resulting second-order differential equation is then given by

$$\frac{\partial^2[V]}{\partial z^2} = [Z][Y][V] \tag{1.25}$$

1.4 Heaviside Systems(6)

In order to solve Eq. (1.25) the matrix product $[\gamma^2] = [Z][Y]$ must be diagonalized. If it is assumed that the solutions are of the form

$$V_i = V_i^f \epsilon^{-az} + V_i^b \epsilon^{+az} \tag{1.26}$$

Eq. (1.25) yields

$$0 = ([\gamma^2] - a^2[\delta_{ij}])[V] \tag{1.27}$$

The propagation constants a_i are therefore the eigenvalues of the propagation matrix $[\gamma^2]$.

For the three-conductor system described previously the eigenvalues are given by

$$a^2 = \frac{\gamma_{11}^2 + \gamma_{22}^2}{2} \pm \left[\gamma_{12}^2\gamma_{21}^2 + \left(\frac{\gamma_{11}^2 - \gamma_{22}^2}{2}\right)^2\right]^{\frac{1}{2}} \tag{1.28}$$

Four different possible propagation constants or two distinct TEM modes result. If the system is symmetric, $\gamma_{22} = \gamma_{11} = \gamma$ and $\gamma_{12} = \gamma_{21} = \gamma_m$, the eigenvalues simplify.

$$a^2 = \gamma^2 \pm \gamma_m^2 \tag{1.29}$$

The two distinct TEM modes will always occur unless γ_m is zero, i.e., the γ^2 matrix is diagonal. This is the case for the lossless system. Equation (1.24) may be solved without difficulty.

$$[V] = [A]\epsilon^{-s\sqrt{\mu\epsilon}\,z} + [B]\epsilon^{s\sqrt{\mu\epsilon}\,z} \tag{1.30}$$

The resulting currents are obtained from Eq. (1.22).

$$[I] = \sqrt{\mu\epsilon}\,[L]^{-1}\left[[A]\epsilon^{-s\sqrt{\mu\epsilon}\,z} - [B]\epsilon^{s\sqrt{\mu\epsilon}\,z}\right] \tag{1.31}$$

If a lossy symmetrical system exists, which can propagate only a single mode, it must have a diagonal propagation matrix. Since the matrix elements must all be equal, the propagation matrix must be of the form

$$[\gamma^2] = f(s)[\delta_{ij}] \tag{1.32}$$

In order to find f(s) the impedance and admittance matrices are expanded.

$$\begin{aligned} [Z] &= [R] + s[L] \\ &= [R] + s\mu\epsilon[p] \end{aligned} \tag{1.33}$$

$$\begin{aligned} [Y] &= [G] + s[C_q] \\ &= [G] + s[p]^{-1} \end{aligned} \tag{1.34}$$

The resulting γ^2 matrix is given by

$$\begin{aligned} [\gamma^2] &= [Z][Y] \\ &= ([R] + s\mu\epsilon[p])([G] + s[p]^{-1}) \end{aligned} \tag{1.35}$$

In order for this expression to be a diagonal matrix [R] and [G] must be restricted.

$$[R] = f_1(s)[p] \tag{1.36}$$

$$[G] = f_2(s)[p]^{-1} \tag{1.37}$$

Then

$$[\gamma^2] = (f_1(s) + s\mu\epsilon)(f_2(s) + s)[\delta_{ij}] \tag{1.38}$$

The eigenvalues are given by

$$a^2 = (f_1(s) + s\mu\epsilon)(f_2(s) + s) \tag{1.39}$$

$$a^2(j\omega) = \alpha^2 - \beta^2\omega^2 + j2\alpha\beta\omega$$

Here α denotes the attenuation constant and β the phase constant. If [R] and [G] are independent of frequency this equation simplifies

$$a^2 = \mu\epsilon(s + \frac{K}{\mu\epsilon})(s + k) \tag{1.40}$$

The restriction

$$k = \frac{K}{\mu\epsilon}$$

results in

$$a = \pm \sqrt{\mu\epsilon}\ (s + \frac{K}{\mu\epsilon}) \tag{1.41}$$

$$a(j\omega) = \pm \left(\frac{K}{\sqrt{\mu\epsilon}} + j\omega \sqrt{\mu\epsilon} \right)$$

Since the group velocity is given by

$$v_g = \left(\frac{\partial\beta}{\partial\omega}\right)^{-1} \tag{1.42}$$

this type of line is nondispersive. It is the generalization of the Heaviside transmission line.

From the preceding arguments it may be concluded that multiconductor transmission line systems may be described by a second-order matrix differential equation. A single TEM mode will be supported if the system is a perfect conductor system imbedded in a lossless dielectric. This is also true if lossy

conductors are considered. However, restrictions on the dielectric losses have to be made, which are related to the conductor losses and the geometry of the system. Both systems possess the property of nondispersive transmission.

2.1 The Symmetric Double-Pair Circuit

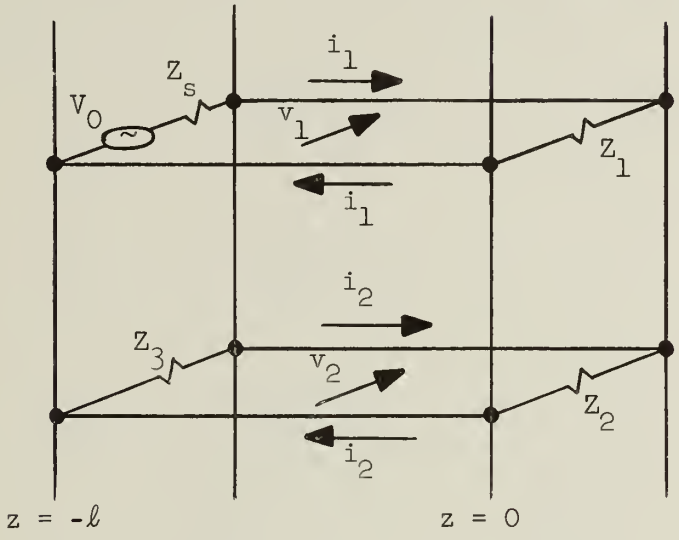

FIGURE 2.1. BOUNDARY CONDITIONS FOR THE SYMMETRICAL DOUBLE-PAIR LINE

The development of Chapter 1 will now be specialized to the balanced double-pair line of Fig. (2.1), or its equivalent: "the two-wire over an infinite ground plane" circuit. This particular topology is described by the expanded forms of Eqs. (1.30) and (1.31).

$$V_i = A_i \epsilon^{-s\sqrt{\mu\epsilon}\, z} + B_i \epsilon^{+s\sqrt{\mu\epsilon}\, z} \qquad i = 1, 2 \tag{2.1}$$

$$I_i = (G_o A_i - G_m A_j)\epsilon^{-s\sqrt{\mu\epsilon}\, z} - (G_o B_i - G_m B_j)\epsilon^{s\sqrt{\mu\epsilon}\, z} \tag{2.2}$$

where

$$i = 1, 2$$

$$j = 2, 1$$

The derivation of the preceding two equations is based on the assumption that the lines are symmetric (see Example 1.1). The line parameters are based on the following definitions.

$$Z_{01} \equiv \sqrt{\frac{L_{11}}{C_{11}}} \tag{2.3}$$

$$Z_{02} \equiv \sqrt{\frac{L_{22}}{C_{22}}}$$

$$Z_{m} \equiv \sqrt{\frac{L_{12}}{C_{12}}}$$

Then

$$G_o \equiv \frac{Z_o}{Z_o^2 - Z_m^2} \qquad G_i = \frac{1}{Z_i} \qquad i = 1, 2, 3, s \tag{2.4}$$

$$G_m \equiv \frac{Z_m}{Z_o^2 - Z_m^2} \qquad b = \sqrt{\mu\epsilon}\,\ell$$

Normalization with respect to G_o will be indicated by the use of lower-case letters.

The integration constants are obtained by evaluating Eqs. (2.1) and (2.2) at the four terminals indicated in Fig. (2.1). The resulting equations are compacted in Eq. (2.5).

$$\begin{bmatrix} V_o \\ 0 \\ 0 \\ 0 \end{bmatrix} = Z_1 Z_2 Z_3 Z_s G_o^4 \begin{bmatrix} (g_s+1)\epsilon^{sb} & -g_m\epsilon^{sb} & (g_s-1)\epsilon^{-sb} & g_m\epsilon^{-sb} \\ -g_m\epsilon^{sb} & (g_3+1)\epsilon^{sb} & g_m\epsilon^{-sb} & (g_3-1)\epsilon^{-sb} \\ g_1-1 & g_m & g_1+1 & -g_m \\ g_m & g_2-1 & -g_m & g_2+1 \end{bmatrix} \cdot \begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \end{bmatrix} \tag{2.5}$$

or

$$\begin{bmatrix} V_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = Z_1 Z_2 Z_3 Z_s G_o^4 \Delta(s) \begin{bmatrix} A_1 \\ A_2 \\ B_1 \\ B_2 \end{bmatrix}$$

The coefficients are therefore given by

$$A_i(s) = \frac{V_o(s)}{G_o Z_s} \frac{\Delta_{1i}(s)}{\Delta(s)} \qquad i = 1, 2$$

$$B_i(s) = \frac{V_o(s)}{G_o Z_s} \frac{\Delta_{1i}(s)}{\Delta(s)} \qquad i = 3, 4 \tag{2.6}$$

If

$$\mathcal{L}^{-1}(A_1(s)) = \mathcal{L}^{-1}\left(\frac{V_o(s)}{G_o Z_s} \frac{\Delta_{11}(s)}{\Delta(s)}\right) \equiv a_1(t) \tag{2.7}$$

and corresponding inverse transforms are defined for the remaining constants; the time domain voltages and currents are given by Eqs. (2.8) and (2.9).

$$v_i(t) = a_i(t - \sqrt{\mu\epsilon}\, z) + b_i(t + \sqrt{\mu\epsilon}\, z) \qquad i = 1, 2 \qquad (2.8)$$

$$i_i(t) = G_o a_i(t - \sqrt{\mu\epsilon}\, z) - G_m a_j(t - \sqrt{\mu\epsilon}\, z) - G_o b_i(t + \sqrt{\mu\epsilon}\, z) + G_m b_j(t + \sqrt{\mu\epsilon}\, z) \qquad (2.9)$$

$$i = 1 \quad \text{and} \quad j = 2 \quad \text{or} \quad i = 2 \quad \text{and} \quad j = 1$$

2.2 Reflectionless Transmission Conditions

In Chapter 1 the conditions for nondispersive transmission were derived. This property is essential for nondistorting, wide-band transmission systems. It is a sufficient condition for truly flat behavior only if the transmission system is infinitely long. However, a finite system must possess the property of reflectionless transmission in order to be a wide-band transmitter. This argument becomes clear if the driving-point impedance of a two-wire transmission line is considered. Only if the line is terminated into its characteristic impedance, i.e., only if reflectionless transmission takes place, does the input impedance become frequency independent. The desirability of this property becomes apparent.

An inspection of Eqs. (2.1) and (2.2) reveals that the suppression of one of the waves may be achieved by equating one of the constants in each of the four equations to zero.

$$\text{(a)} \quad A_1 = B_2 = 0 \quad \text{or} \quad A_2 = B_1 = 0$$

$$\text{(b)} \quad A_1 = B_1 = 0 \quad \text{or} \quad A_2 = B_2 = 0 \qquad (2.10)$$

$$\text{(c)} \quad A_1 = A_2 = 0 \quad \text{or} \quad B_1 = B_2 = 0$$

Condition (a) results in a voltage match only. Condition (b) is trivial. Only the last case of condition (c) remains of interest for the choice of coordinates indicated in Fig. 2.1. Hence, by referring to Eq. (2.6), it appears that the desired condition is given by:

$$\Delta_{14} = \Delta_{13} = 0 \tag{2.11}$$

$$\Delta \neq 0$$

If use is made of the fact that a determinant is zero when two rows or columns are equal to each other, the termination condition (2.11) is satisfied if the ratio of column 1 to column 2 is taken in Eq. (5.2).

$$\frac{-g_m}{g_3 + 1} = \frac{g_1 - 1}{g_m} = \frac{g_m}{g_2 - 1} \equiv -K \tag{2.12}$$

That this is the only meaningful solution may be seen from the following argument. Expansion of Eq. (2.11) yields:

$$g_m \epsilon^{-sb} \Delta_{12,34} - (g_1 + 1)\Delta_{13,34} - g_m \Delta_{14,34} = 0 \tag{2.13}$$

$$(g_3 - 1)\epsilon^{-sb} \Delta_{12,34} - g_m \Delta_{13,34} + (g_2 + 1)\Delta_{14,34} = 0$$

The a priori selection of two of the four coefficients results in two dependent termination conditions. This means that only Z_s and one of the three remaining terminations may be selected arbitrarily, but Z_o and Z_m remain independent. Therefore, Eq. (2.11) can only be satisfied if all second-order minors vanish. But this condition is equivalent to Eq. (2.12). The termination relationships for reflectionless transmission are:

$$G_2 = -G_3 \tag{2.14}$$

$$G_1 = \frac{G_o G_2 - (G_o^2 - G_m^2)}{G_2 - G_o} \tag{2.15}$$

It is important to note that

(1) the transmission property was established for the source condition indicated in Fig. (2.1) only.

(2) the imposition of additional restrictions due to reflectionless transmission requirements from any of the remaining ends will result in a vanishing of the determinant and, therefore, yield an oscillatory solution.

These arguments may be extended to multiconductor systems. Therefore, it follows that: "Any n-conductor, completely parallel, closed, nondispersive transmission line system can be matched at most from one driving port for reflectionless transmission in the same direction on all pairs." This theorem negates the possibility of finding a distributed transformer with more than one input port from which the system appears to be terminated as desired.

2.3 Allowed Terminations for Matched Conditions

The types of terminations which are allowed may be obtained by considering the implications of Eq. (2.14). Since

$$Z_3 = -Z_2$$

it is obvious that one of the terminations cannot be positive real if the terminations are not purely imaginary. If only reactive elements are involved,

the above equation expresses a resonance condition and is therefore satisfied only for certain frequencies.

(a) Single-frequency steady state matching:

$$-G_2 = +G_3$$

$$\frac{1}{sL_2} = sC_3 \qquad s = j\omega_o \tag{2.16}$$

The termination impedance of line #1 is given by Eq. (2.13).

$$Z_1 = \frac{Z_o + sC_3(Z_o^2 - Z_m^2)}{1 + sC_3Z_o} \tag{2.17}$$

$$Z_1 = \frac{Z_o}{1 + sC_3Z_o} + \frac{sC_3(Z_o^2 - Z_m^2)}{1 + \dfrac{sC_3(Z_o^2 - Z_m^2)}{\dfrac{Z_o^2 - Z_m^2}{Z_o}}}$$

This impedance is positive real. Its realization is indicated in Fig. 2.2.

If the secondary terminations are interchanged a different result occurs.

$$sC_2 = \frac{1}{sL_3} \tag{2.18}$$

The primary termination is now given by:

$$Z_1 = \frac{Z_o}{1 - sC_2Z_o} - \frac{sC_2(Z_o^2 - Z_m^2)}{1 - \frac{sC_2(Z_o^2 - Z_m^2)}{\frac{Z_o^2 - Z_m^2}{Z_o}}} \qquad (2.19)$$

This impedance is not positive real and, therefore, cannot be realized by passive elements.

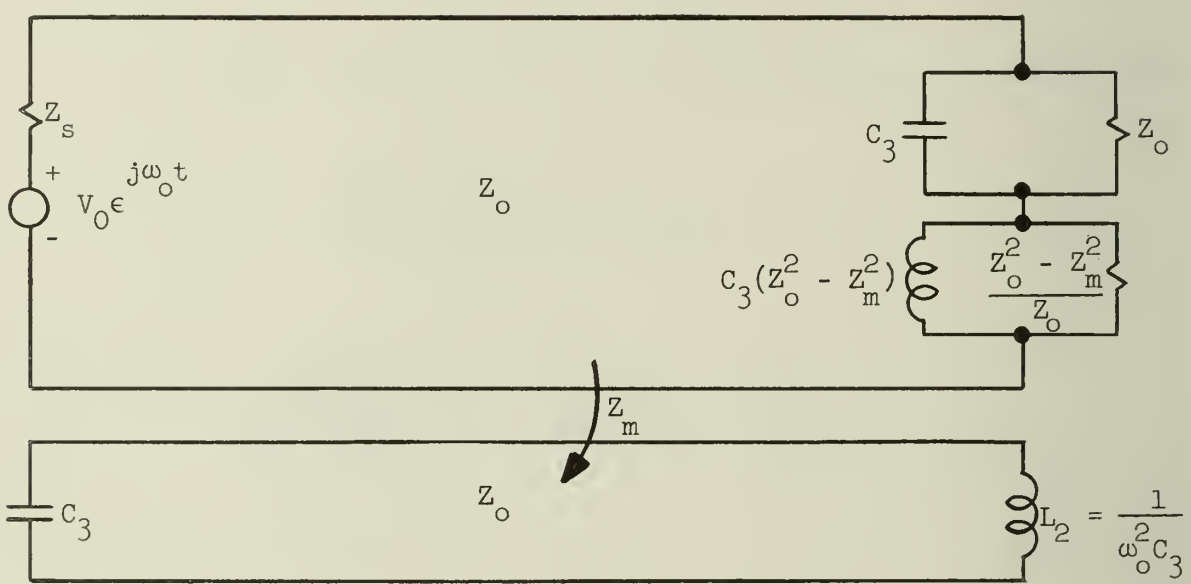

FIGURE 2.2. SINGLE-FREQUENCY STEADY-STATE MATCHING

(b) Frequency-Independent Matching:

The assumption that either Z_2 or Z_3 is positive real may be used to define the terminations more generally. Since:

$$Z_{pr} = \frac{\Delta^L}{\Delta^L_{jj}} \qquad (2.20)$$

where Δ^L and Δ^L_{jj} refer to the loop current determinant of the Z_{pr} complex, the impedance

$$Z' = \frac{(-1)^n}{(-1)^{n-1}} \frac{\Delta^L}{\Delta^L_{jj}} = -Z_{pr} \tag{2.21}$$

is the topologically-similar negative of Z_{pr}. It includes only negative elements. If negative reactances are excluded, Z' is a network of negative resistors. For this condition the sending-end input impedance becomes real. It is this condition which offers frequency-independent matching and, therefore, may have the desired properties of the ideal pulse transformer. These matching conditions are portrayed in Fig. 2.3. It is seen from this drawing that a classification according to the signs of the termination resistors is possible (see also Table 2.1).

TABLE 2.1. CLASSIFICATION OF THE MATCHING CONDITIONS FOR THE RESISTANCE-TERMINATED IDEAL CASE

Class	G_3	G_1	G_2
I	$0 < G_3 < \infty$	$G_o > G_1 > 0$	$-\infty < G_2 < 0$
II	$-\frac{G_o^2 - G_m^2}{G_o} < G_3 < 0$	$G_o > G_1 > 0$	$\frac{G_o^2 - G_m^2}{G_o} > G_2 > 0$
III	$-\infty < G_3 < -G_o$	$\infty > G_1 > G_o$	$\infty > G_2 > G_o$
IV	$-G_o < G_3 < -\frac{G_o^2 - G_m^2}{G_o}$	$-\infty < G_1 < 0$	$G_o > G_2 > \frac{G_o^2 - G_m^2}{G_o}$

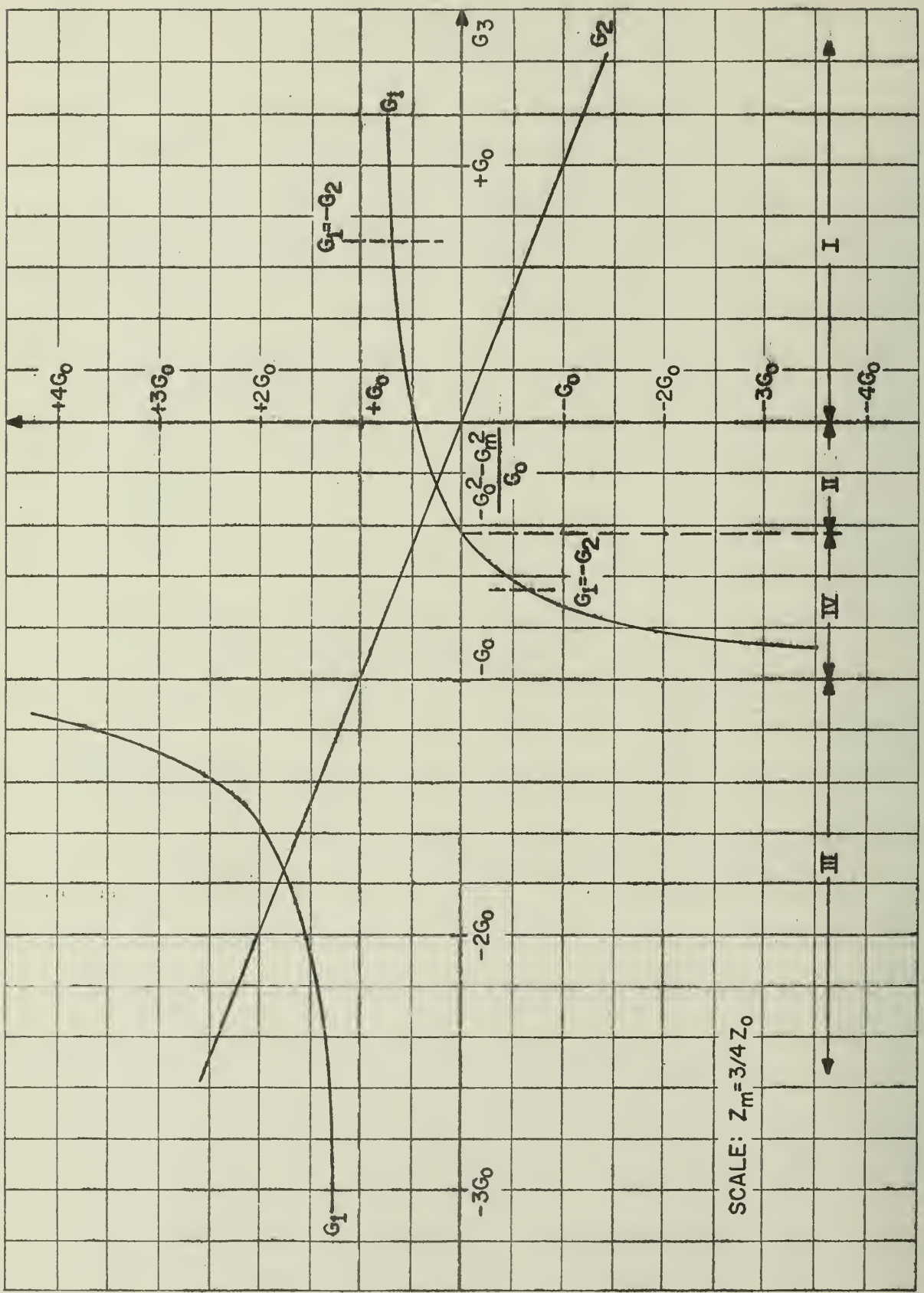

FIGURE 2-3: MATCHING CONDITIONS FOR THE RESISTANCE TERMINATED IDEAL CASE.

2.4 Properties of the Resistance-Terminated Case

If a voltage step of magnitude V_o is applied to the ideally terminated case, the currents and voltages on the two lines are given by the following set of equations.

$$v_1 = \frac{Z_1}{Z_1 + Z_s} V_o U(t - \sqrt{\mu\epsilon}\,(\ell + z)) \tag{2.22}$$

$$v_2 = + \frac{Z_1 K}{Z_1 + Z_s} V_o U(t - \sqrt{\mu\epsilon}\,(\ell + z))$$

$$i_1 = \frac{Z_1(Z_o - Z_m K)}{(Z_1 + Z_s)(Z_o^2 - Z_m^2)} V_o U(t - \sqrt{\mu\epsilon}\,(\ell + z)) \tag{2.23}$$

$$i_2 = \frac{+Z_1(Z_o K - Z_m)}{(Z_1 + Z_s)(Z_o^2 - Z_m^2)} V_o U(t - \sqrt{\mu\epsilon}\,(\ell + z))$$

where

$$U(t - \sqrt{\mu\epsilon}\,(\ell + z)) = 1 \qquad t > \sqrt{\mu\epsilon}\,(\ell + z)$$

$$= 0 \qquad t < \sqrt{\mu\epsilon}\,(\ell + z)$$

The impedances in the direction of propagation are given by:

$$Z_s^{(1)} \equiv \frac{v_1}{i_1} = Z_1 \tag{2.24}$$

and

$$Z_s^{(2)} \equiv \frac{v_2}{i_2} = Z_2 \tag{2.25}$$

The impedance $Z_s^{(2)}$ is to be measured with the sending end of line #1 excited. The physical meaning of the constant of proportionality K becomes apparent if the ratio of the line voltages is considered.

$$K = \frac{v_2(z,t)}{v_1(z,t)} \tag{2.26}$$

A similar relationship may be derived for the current transfer.

$$\frac{i_2(z,t)}{i_1(z,t)} = -K \frac{Z_1}{Z_3} \equiv K_i \tag{2.27}$$

A power transfer relationship is very difficult to define. The network has essentially three useable outputs. The true power output should therefore be the sum of these three termination powers. However, the outputs do not occur in time phase and cannot be connected together in order to drive a single load from the same driving point if DC-isolation is insisted upon. Furthermore, the driving-point impedance of the coupled section may become negative. It is therefore perhaps best to state that power gain does occur, if the magnitude of the voltage transfer exceeds unity, and the magnitude of the input impedance exceeds that of the secondary termination impedance. This type of definition unfortunately does not include the generator impedance. If the available input voltage has some upper limit V_u, and if it is necessary to obtain a minimum output voltage of V_{min}, then Eq. (2.28) must be satisfied.

$$\frac{V_u Z_s^{(1)}}{Z_s + Z_s^{(1)}} |K| > V_{min} \tag{2.28}$$

The generator impedance does therefore have an upper bound.

$$Z_s < Z_s^{(1)}(|K| \frac{V_u}{V_{min}} - 1) \tag{2.29}$$

Therefore, even if $K > 1$ and the secondary impedance is lower than the primary impedance, i.e., power gain does occur, unless the generator impedance can be made low enough, such a circuit cannot satisfy the requirements as a power amplifier. Since the upper limit V_u is not only a function of the active element used in the driving circuit but also of the topology of that circuit, a generalization of power gain in terms of this parameter seems impossible. A possible solution to this problem, but by no means a completely satisfactory one, is offered by the following relationships.

$$P_{in} \equiv i_1^2(R_s + R_1) \tag{2.30}$$

$$P_{out} \equiv i_2^2|R_3|$$

Then:

$$K_p \equiv \frac{P_{out}}{P_{in}} = (K_i)^2 \frac{|R_3|}{R_1 + R_s} \tag{2.31}$$

If the system is power matched from the source, i.e., $R_1 = R_s$, the power transfer is given by:

$$K_{p_{max}} = \frac{1}{2} |KK_i| \tag{2.32}$$

These power transfer relationships will not be used to decide whether a particular combination of terminations will meet a requirement or not. Rather they will be used in the sense of indicators and as aids in the evaluation of a given set of parameters.

The transfer relationships are described in Fig. 2.4. Region I is very similar to a passive termination. The secondary terminations must be low impedances in order to get any sizeable output on the secondary. The voltage transfer is noninverting. Power gain seems impossible. Region II has nearly the opposite properties of region I, with the exception that the voltage transfer is also noninverting. Region III contains the conditions for a low impedance secondary with inverted voltage output and the possibility of power gain. Region IV utilizes two negative terminations. It indicates the possibility of high power gain with noninverted output. However, G_1 varies rapidly in this region, so that very accurate control over the circuit parameters must be exercised. Nevertheless, the conclusion must be that for the ideal case voltage amplification, both inverting and noninverting, is possible. Some form of power amplification seems to be possible.

The arguments presented in this chapter permit the following conclusions. Restricted combinations of positive and negative resistors may be found, which, when used as terminations for two nondispersive, intercoupled transmission lines, yield an over-all transmission system, which is characterized by reflectionless transmission on both lines, when these are excited from one of the four ports of the network. The allowed termination values, which produce this type of transmission, form a continuum. This is in contrast to the single transmission line, where this type of transfer can be obtained only for a single value of the termination impedance. Properties associated with a system terminated in this fashion are: resistive input impedance, voltage and current transfers which may exceed unity and DC-isolation between input and output.

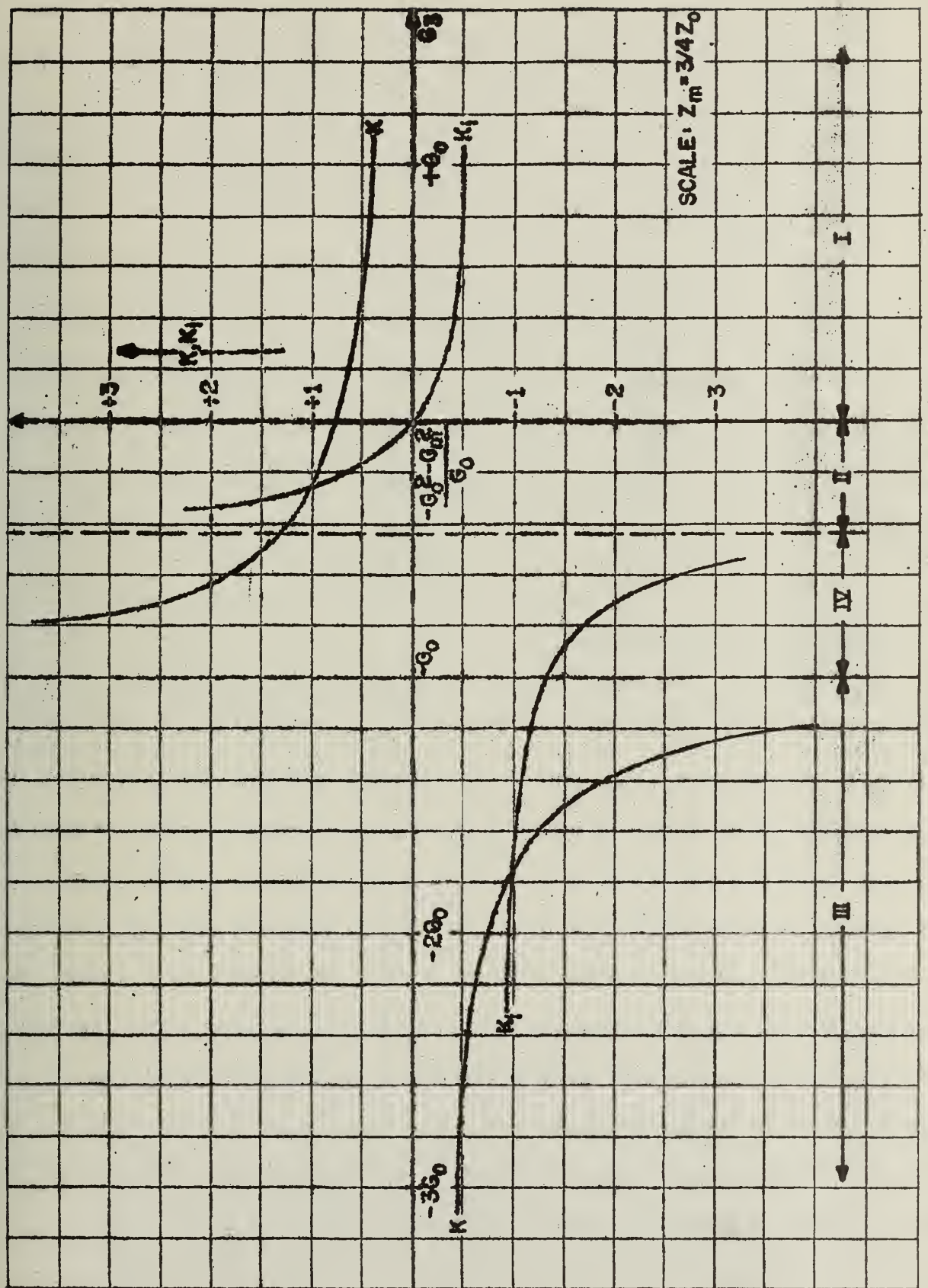

FIGURE 2-4 VOLTAGE AND CURRENT TRANSFER PROPERTIES FOR THE IDEAL RESISTANCE TERMINATED CASE.

3. STABILITY CONDITIONS

The lossless, infinitely long mathematical model postulated in the first part of Chapter 1, has been changed to a theoretical model which may involve losses, has finite physical dimensions and uses positive and negative resistors for its terminations. It is based on the assumption that

(1) negative, constant resistors exist.

(2) a suitable transmission line system can be constructed.

(3) the entire system is stable.

Points (1) and (3) depend upon each other if the negative resistors used are associated with nonremovable reactances. In order to study the effect of these inaccessible parameters, a general stability theory for the proposed system is needed.

3.1 Active Circuit Representation

A number of theorems exist which treat the stability behavior of circuits consisting of short-circuit stable and open-circuit stable structures. However, the conversion problem from an arbitrary to either a short-circuit stable or open-circuit stable immittance is in general avoided. This is partly a consequence of the use of the term physical realizability. Bode[1] treats linear active networks and defines realizability be demanding that the circuit be not self-destructing. However, a linear active theory involves constant negative resistors and, therefore, involves apparently infinite energy sources. A network, which, in the light of this theory, is in fact self-destroying is certainly not so, if saturation effects are accounted for or if this type of theory is restricted to a region of responses and excitations. If a network is defined as physically nonrealizable when it is neither open-circuit nor short-circuit stable, other difficulties appear. The addition of some

immittance to the network produces a total network which is stable. But this total network includes a network which was physically nonrealizable. A theory based on this definition must have difficulties in the establishment of permissible basic building blocks. However, it does serve here to point out the more basic problem of finding a stabilizing immittance, which must be permitted to be an active network. Physical realizability may be looked upon as the solution to the synthesis problem of finding those immittances which have to be added to a given network in order to produce a total system which is either short- or open-circuit stable or both if energized from prescribed terminals.

The active circuit may be described in terms of the six elements $\pm$ R, $\pm$ L and $\pm$ C. The description is redundant in the sense that the elements -L and -C are derivable from the elements $\pm$ R, +L and +C. This follows from the circuits shown in Fig. 3.1. These four elements may be described in terms of the single circuit of Fig. 3.2. The proof of this statement is found in Table 3.1. The transition from the four elements to the basic active circuits indicates that the allowed values of Z_1 and Z_2 are not uniquely determined.

TABLE 3.1

Z_{in}	Z_1	Z_2
+R	2R	∞
-R	0	∞
sL	R + sL	∞
-sL	R	$R + \frac{1}{s(\frac{L}{R^2})}$
$\frac{1}{sC}$	$R + \frac{1}{sC}$	∞
$-\frac{1}{sC}$	R	$R + s(CR^2)$

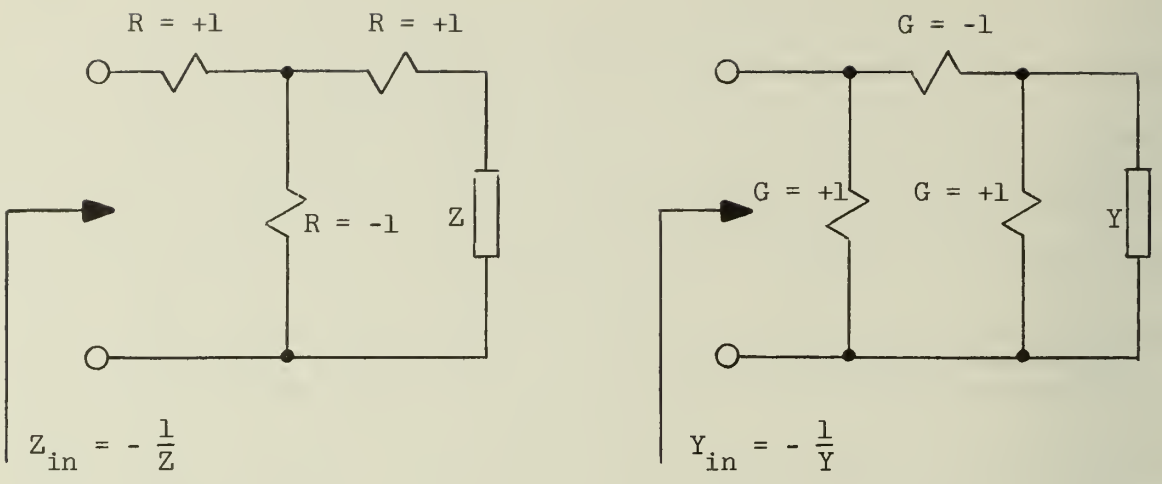

FIGURE 3.1. NEGATIVE INVERSE CIRCUITS

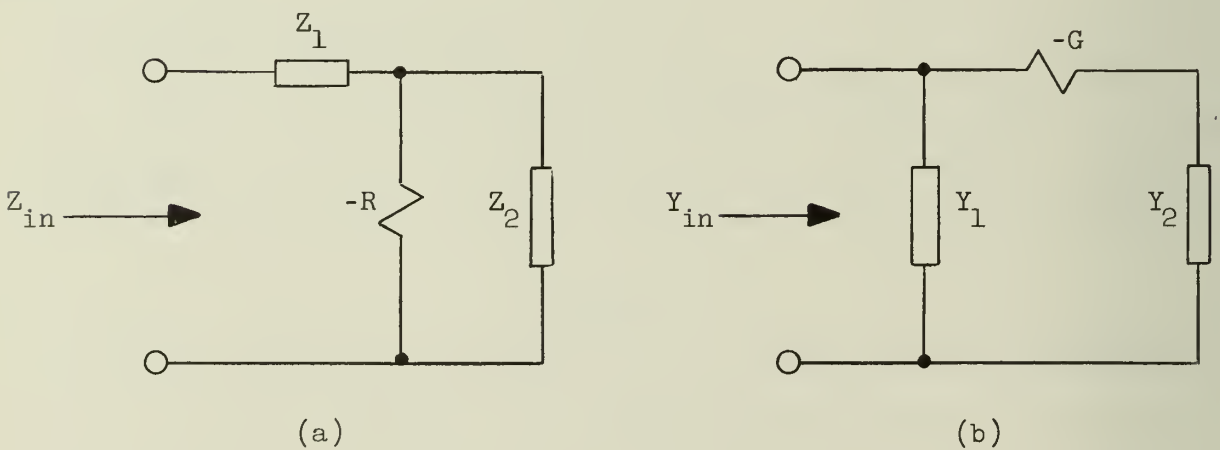

FIGURE 3.2. BASIC ACTIVE CIRCUITS

Other forms of Z_1 and Z_2 may be found to simulate the negative elements. However, in all cases Z_1 and Z_2 are positive real, i.e., they are formed from passive elements. The conclusion is that the active circuit may be represented in terms of the basic active circuit of Fig. 3.2a or its dual, indicated in Fig. 3.2b.

If an open (short) circuit stable admittance (impedance) is defined as one which has no right half plane zeros, the four basic elements $\pm$ R, L and C are both open- and short-circuit stable. Stability problems result from their interconnection. Hence, stability is in this sense a circuit rather than an element property. The occurrence of physically inaccessible nodes in the equivalent circuit of the active element may be used to call it a device property. However, the equivalent may be reduced to a description in terms of basic active circuits. The occurrence of inaccessible nodes is expressible as constraints on Z_1 or Z_2 or both. Therefore, stability may be treated as the property of the basic active circuit subject to a set of suitable interconnection rules.

3.2 The Basic Active Circuit

The input impedance of the circuit of Fig. 3.2a is given by:

$$Z_{in} = (Z_1 - R) - \frac{R^2}{(Z_2 - R)} \qquad (3.1)$$

$$= Z_1^1 - \frac{R^2}{Z_2^1}$$

It consists of the difference of Z_1^1 and the inversion of Z_2^1 in the circle of radius R. Equation (3.1) may be rewritten to yield:

$$Z_{in} = \frac{\frac{Z_1 Z_2}{Z_1 + Z_2} - R}{\frac{Z_2 - R}{Z_1 + Z_2}} \tag{3.2}$$

Since Z_1 and Z_2 were assumed to be positive real, the denominator of the preceding equation cannot have any right half plane poles. Therefore, if Z_{in} is to be short-circuit stable it is sufficient to restrict the zeros of the numerator of Eq. (3.2) to the left half plane.

Hence,

$$N(s) = \frac{Z_1 Z_2}{Z_1 + Z_2} - R \tag{3.3}$$

must be a Hurwitz polynomial. Furthermore, since this expression is invariant to an interchange of Z_1 and Z_2, it follows that, if Z_{in} is short-circuit stable, then Z'_{in} is also short-circuit stable.

$$Z'_{in} = (Z_2 - R) - \frac{R^2}{(Z_1 - R)} \tag{3.4}$$

Extension of these arguments to the circuit of Fig. 3.2b indicate similar conditions. The preceding statements apply if short-circuit stable is replaced by open circuit stable and impedance by admittance.

Equation (3.3) may be expressed as

$$N(s) = Y_1 + Y_2 - G = 0 = Y - G \tag{3.5}$$

where

$$Y_1 = \frac{1}{Z_1}, \quad Y_2 = \frac{1}{Z_2}, \quad G = \frac{1}{R}$$

if

$$Y \equiv \frac{P(s)}{Q(s)}$$

The stability condition may be restated by requiring R(s) in Eq. 3.6 to be a Hurwitz polynomial.

$$R(s) = \pm (P(s) - GQ(s)) \tag{3.6}$$

The meaning of the double sign may be seen from the following argument. Since R(s) is Hurwitz all coefficients have the same sign. Furthermore, since Z_1 and Z_2 were assumed to be positive real, i.e., they are formed from passive elements, the orders of P(s) and Q(s) can differ at most by unity. Therefore, only a single Hurwitz polynomial is possible if the orders differ. However, if the orders are the same two solutions result.

$$R'(s) = P(s) - GQ(s) \tag{3.6a}$$

$$R''(s) = GQ(s) - P(s) \tag{3.6b}$$

Example 3.1. Tunnel Diode

$$Y_2 = sC$$

$$Y_1 = \frac{1}{R_s + sL}$$

$$R(s) = s^2LC + s(R_sC - GL) + (1 - GR_s)$$

This is a Hurwitz polynomial if all coefficients are positive. Therefore, the stability conditions are:

$$1 - GR_s > 0$$

$$R_sC - GL > 0$$

or

$$R > R_s > \frac{L}{RC}$$

where

$$R_s = r + r_s$$

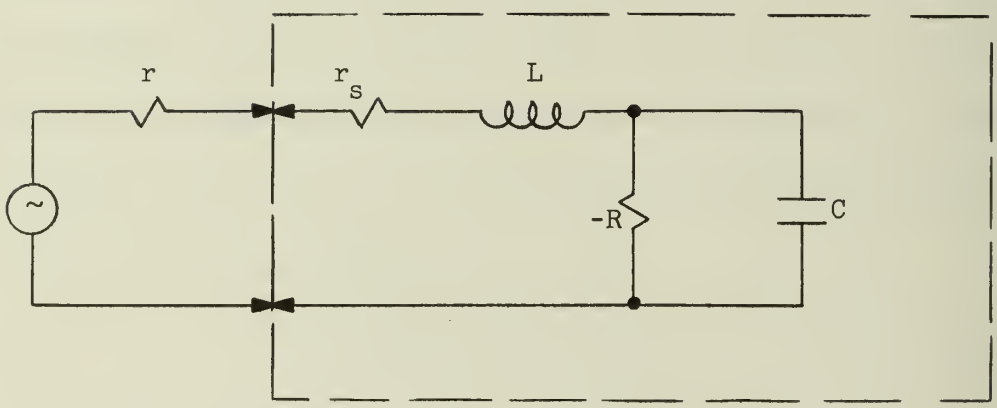

FIGURE 3.3 TUNNEL DIODE STABILIZATION

The network can be made short-circuit stable if r is selected in such a way that the last condition is satisfied.

If it is assumed that Y_1 and Y_2 are reactance functions their sum is also a reactance function. Therefore, it is the ratio of an even to an odd or an odd to an even Hurwitz polynomial. The polynomial R(s) of Eq. (3.6) can never be Hurwitz. Therefore, in order to make the basic network short-circuit stable, either Y_1 or Y_2 or both must contain dissipative elements.

On the other hand, if the minimum value of $\mathcal{R}(Y(j\omega))$ exceeds G, and, consequently $\mathcal{R}(Y(s)) \geq G$ for $\mathcal{R}(s) \geq 0$, it is possible to use a resistance reduction on Y. Hence,

$$Y = (Y - g) + g$$

$$g \geq G$$

$$Y - G = (Y - G) + (g - G)$$

and (Y - G) is again a positive real admittance. The resulting R(s) is Hurwitz. Therefore, this type of system is always short-circuit stable.

The more general solution is bounded by these two cases. It may be obtained by making use of the Nyquist criterion. Since Y(s) is positive real its Nyquist plot must be in the right half $Y(j\omega)$ plane. Furthermore, the plot of Y - G cannot encircle the origin of this plane.

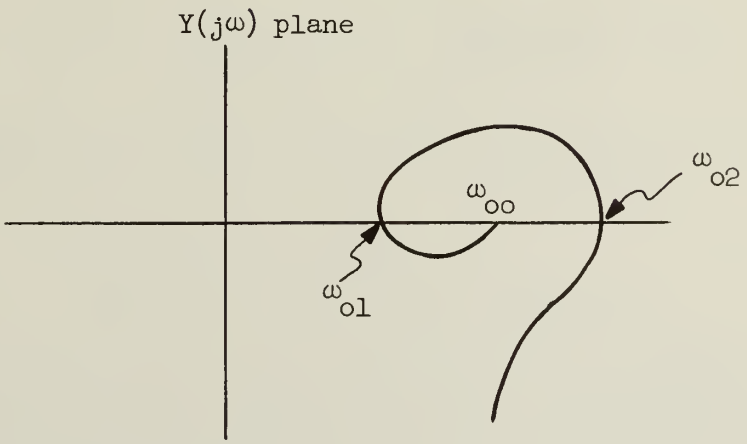

FIGURE 3.4. POSITIVE REAL FUNCTION ($\omega \geq 0$)

The self-resonant frequencies of the admittance Y are given by the solutions of the equation:

$$\mathrm{Im}(Y(j\omega)) = 0 \tag{3.7}$$

These frequencies are denoted by:

$$0 < \omega_{o1} < \omega_{o2} \ldots < \omega_{oi}$$

Substitution of the resonances into the real part of Y yields:

$$\mathcal{R}(Y(j\omega_{oj})) = F(\omega_{oj}) \qquad j = 1, 2, \ldots, i \tag{3.8}$$

These quantities can again be ordered according to magnitude. Their maximum value is denoted by $F(\omega_{oU})$ and their minimum by $F(\omega_{oL})$. The sufficient conditions for stability are then expressed by Eq. (3.9).

$$\begin{aligned} &\text{(a)} \quad G < F(\omega_{oL}) \\ &\text{(b)} \quad G > F(\omega_{oU}) \end{aligned} \tag{3.9}$$

The unstable conditions are therefore located in the range:

$$F(\omega_{oU}) > G > F(\omega_{oL}) \tag{3.10}$$

These solutions may be correlated with the corresponding solutions in terms of the Hurwitz criterion. If the admittance Y has a zero at infinity $F(\omega_{oL}) = 0$, so that Eq. (3.9a) cannot be satisfied. If Y has a pole at infinity $F(\omega_{oU}) \to \infty$, hence Eq. (3.9b) cannot be satisfied. However, if $F(\omega_{oL}) \neq 0$ and $F(\omega_{oU}) < \infty$ Eq. (3.9) indicates the possibility of two solutions, i.e., it leads to the same result as Eqs. (3.6a, b). Of course, in practical cases Y will always have a pole at infinity. Equation (3.9b) is therefore somewhat academic.

The physical implication of Eq. (3.3) may be interpreted in another way. The system determinant, i.e., the description of the basic circuit with all external sources removed and replaced by open or short circuits, is given by:

$$\Delta_y = Y_1 + Y_2 - G \tag{3.11}$$

The stability criterion based on the roots of the system determinant in the s plane is equivalent to Eq. (3.3). This implies that the requirement for a system to be stable is related to the self-resonances and the resistive cutoff frequencies of the system. The relationship may be deduced from Fig. (3.5).

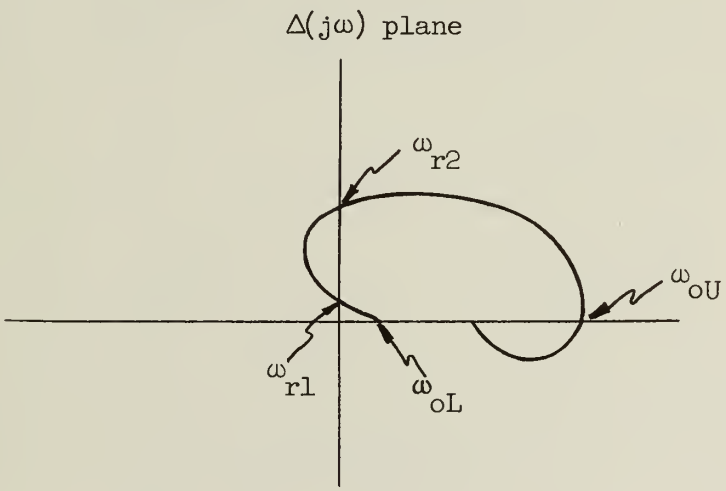

FIGURE 3.5. RESONANCES OF THE SYSTEM DETERMINANT

It is easily seen that the system is stable only if all self-resonances occur in the right half-plane of $\Delta(j\omega)$. Furthermore, if resistive cutoffs do occur and are denoted by ω_{rn}^2, n = 1, 2, ..., j, then j must be an even integer and $\omega_{rj}^2 \neq \omega_{ri}^2$ for all i and j.

3.3 Nonlinear Models

The preceding results apply only to linear models. Extension to nonlinear models is possible if these are approximated by a number of linear models. The assumption that a fixed linear negative resistor exists is to be replaced by the assumption that a variable, current and voltage-independent resistor exists. In other words, G, the magnitude of the negative conductance is specified by:

$$G_{min} \leq G \leq G_{max} \tag{3.12}$$

Unconditional short-circuit stability is defined by the requirement that the system is to be short-circuit stable for all values of G in this range.

With these restrictions the stability conditions of Eq. (3.9) become:

$$\begin{aligned} &(a) \quad F(\omega_{oL}) > G_{max} \\ &(b) \quad F(\omega_{oU}) < G_{min} \end{aligned} \tag{3.13}$$

But, by definition

$$F(\omega_{oL}) < F(\omega_{oU})$$

Therefore, no conductance exists which has unconditional short-circuit stability in both regions of Eq. (3.13). However, it becomes also apparent that conductances can be found which are stable in one of the two regions. In particular, if

$$0 \leq G \leq G_{max} \tag{3.14}$$

and

$$F(\omega_{oL}) > G_{max}$$

where

$$G = |-G|$$

the system is unconditionally short-circuit stable in the region specified by Eq. (3.13a). On the other hand, if

$$G_{min} \leq G < \infty \tag{3.15}$$

and

$$F(\omega_{oU}) < G_{min}$$

a similar statement holds for the region of Eq. (3.13b).

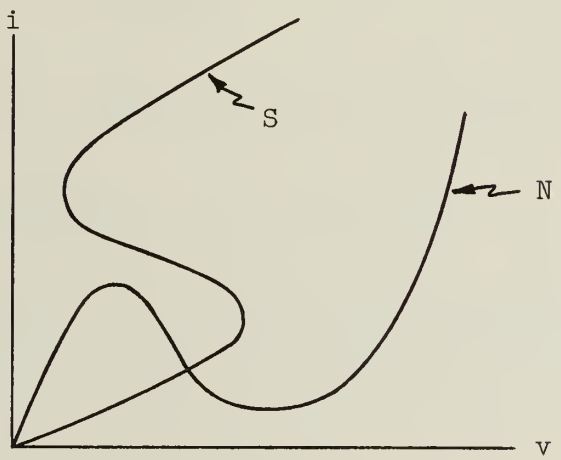

FIGURE 3.6. S- AND N-TYPE NONLINEARITIES

Association of Eq. (3.14) with the N-type nonlinearity and of Eq. (3.15) with the S-type nonlinearity leads to the conclusion that both S- and N-type resistors can be made short-circuit stable by an admittance which has neither pole nor zero at infinity. If the admittance has a pole at infinity its association with an S-type element cannot be short-circuit stable. However, physical admittances always have a pole at infinity due to stray capacitances.

Hence, an S-type element can only be made short-circuit stable by the use of ideal elements. If the admittance has a zero at infinity the N-type element cannot be made short-circuit stable.

The development thus far is based on the supposition that a negative resistor is available. Energy considerations restrict its occurrence to a range on the v-i characteristic of the element and to an incremental nature. The possibility of biasing in this region is implied. It is necessary to investigate whether the condition that the circuit is stable for $\omega = 0$ can be interpreted as a guarantee that a bias point can be established.

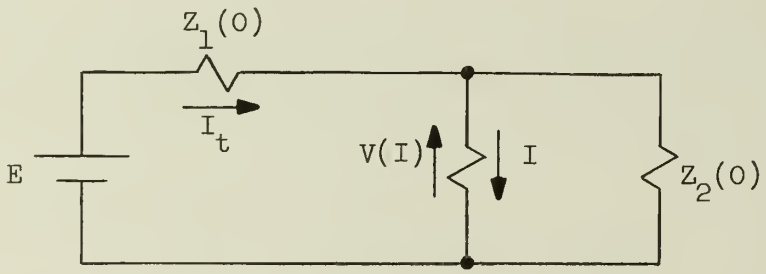

FIGURE 3.7. BIAS CONDITIONS

A slight change in the bias E of Fig. 3.7 will result in the following relationships:

$$\frac{\Delta E}{\left(1 + \frac{1}{Z_2(0)} \frac{\Delta V}{\Delta I}\right)} = \left(Z_1(0) + \frac{\frac{\Delta V}{\Delta E} Z_2(0)}{\frac{\Delta V}{\Delta I} + Z_2(0)}\right) \Delta I \qquad (3.16)$$

or

$$\frac{\Delta E}{\Delta I_t} = Z_1(0) + \frac{\frac{\Delta V}{\Delta I} Z_2(0)}{\frac{\Delta V}{\Delta I} + Z_2(0)} \qquad (3.17)$$

The assumption that the left side of Eq. (3.16) is finite can be valid only if the right side is also finite. Hence, if

$$Z_1(0) + \frac{\frac{\Delta V}{\Delta I} Z_2(0)}{\frac{\Delta V}{\Delta I} + Z_2(0)} = 0 \tag{3.18}$$

ΔI must become infinite. But this means that the bias point cannot be established. Equation (3.17) indicates that this condition is equivalent to

$$Z_{in}(0) = 0$$

or, by the same argument which resulted in Eq. (3.6):

$$Y_1(0) + Y_2(0) + \frac{\Delta I}{\Delta V} = 0 \tag{3.19}$$

But the N-type stability condition of Eq. (3.14) may be written as:

$$Y_1(0) + Y_2(0) \geq F(\omega_{oL}) > G_{max} \tag{3.20}$$

This implies that the switching condition, Eq. (3.19) cannot be satisfied. A similar argument may be applied to the S-type element. This yields:

$$Y_1(0) + Y_2(0) \leq F(\omega_{oU}) < G_{min} \tag{3.21}$$

Therefore, the conclusion is that the stability conditions do allow for biasing and that stability including $\omega = 0$ may be interpreted to imply this possibility.

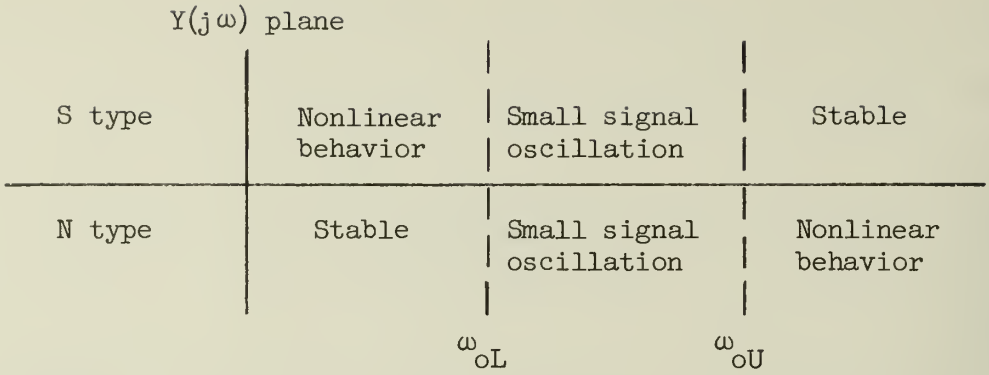

FIGURE 3.7. SHORT-CIRCUIT STABILITY OF N- AND S-TYPE ELEMENTS IN PARALLEL WITH AN ADMITTANCE WHICH HAS NEITHER POLE NOR ZERO AT INFINITE FREQUENCY

3.4 Interconnection Rules

The allowed methods of interconnection may be based partly on Bode's theorem.[1] A short-circuit stable impedance is again short-circuit stable if connected in parallel with another short-circuit stable impedance. A similar theorem exists for the open-circuit stable case. These two theorems allow interconnection between elements of the same type (Figs. 3.2a, b). Since the general, linear active circuit may be described completely in terms of only one of these circuits, the stability problem must be considered solved in principle. The conclusion that both types of nonlinearities may be made short-circuit stable by idealized admittances, allows the simultaneous use of both circuits for the description of a single system.

However, more flexibility is obtained if it is considered that, if the loop impedance of Fig. 3.2a is short-circuit stable, then the shunt admittance of the metwork which remains when the input terminals are shorted, has neither poles nor zeros in the right half-plane. This admittance is open-circuit stable. Then the short-circuit stable network may be connected to the open-circuit stable network as shown in Fig. 3.8.

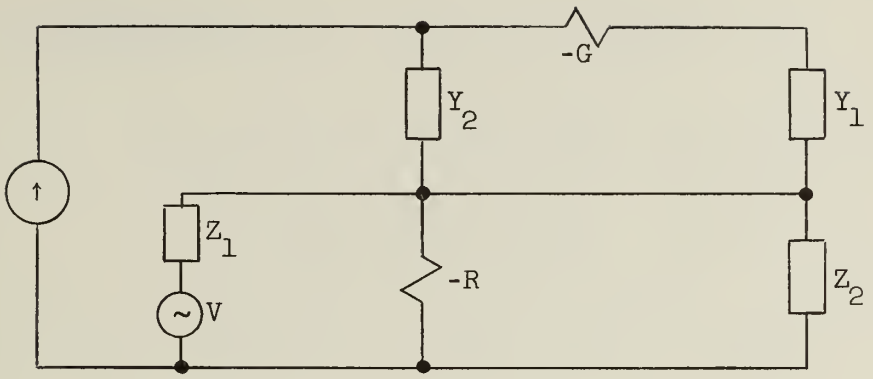

FIGURE 3.8. OPEN-CIRCUIT STABLE INTERCONNECTION OF THE BASIC ACTIVE CIRCUIT AND ITS DUAL

Example 3.2. Tunnel Diode Transmission Line Stability

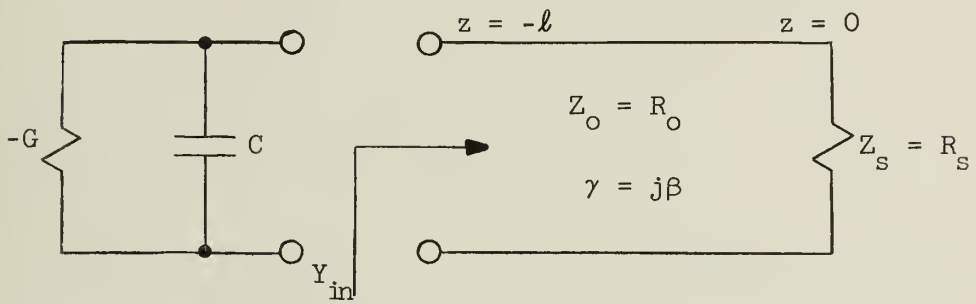

FIGURE 3.9. LOSSLESS RESISTIVELY TERMINATED TRANSMISSION LINE LOADED BY TUNNEL DIODE

The input admittance of the line is given by:

$$Y_{in} = Y_o \frac{1 - q\epsilon^{j2\beta\ell}}{1 + q\epsilon^{j2\beta\ell}} \tag{3.22}$$

where

$$q = \frac{Z_s - Z_o}{Z_s + Z_o} \tag{3.23}$$

This quantity is assumed to be real. Hence,

$$Y_{in} = \frac{1 - q^2}{1 + q^2 + 2q \cos 2\beta\ell} - j \frac{2q \sin 2\beta\ell}{1 + q^2 + 2q \cos 2\beta\ell}$$

The stability condition is based on Eq. (3.14) and becomes

$$G_{max} < (\mathcal{R}e(Y_{in}))_{min} \tag{3.24}$$

or

$$G_{max} < Y_o \frac{1 + q}{1 - q} = Y_o^2 Z_s$$

$$Z_o^2 < R_{min} Z_s \tag{3.25}$$

If

$$Z_s \equiv Z_o(1 - \Delta)$$

the last expression becomes

$$Z_o < R_{min}(1 - \Delta)$$

However, the bias conditions are given by

$$Z_s = Z_o(1 - \Delta) < R_{min}$$

Stability for all cases is assured if

$$R_{min} > Z_o \tag{3.26}$$

3.5 Stability Conditions for the Double-Pair Transformer with Ideal Negative Resistance Terminations

In the preceding chapter the properties of the resistance terminated ideal case were discussed. If such a system is to be meaningful it must of course be stable. Since resistors of both signs are allowed, difficulties due to unfortunate choices of termination values must be expected. These will be frequency independent. In addition the normal frequency dependent stability conditions must occur. Since, for the ideal case, two of the four coefficients are always zero, Δ must be restricted to nonzero values. Hence, Eq. (3.27) cannot be satisfied.

$$\Delta = -2Z_s Z_1 Z_2 Z_3 \frac{G_o G_m}{K} (G_s + G_1)(G_1 + G_2)(1 - \epsilon^{s\sqrt{\mu\epsilon}\, 2\ell}) = 0 \qquad (3.27)$$

The prohibited termination values are:

$$\text{(a)} \quad G_s = -G_1$$

$$\text{(b)} \quad G_1 = -G_2 \quad \text{or} \quad R_2 = \pm\sqrt{Z_o^2 - Z_m^2} \qquad (3.28)$$

$$\text{(c)} \quad K = \infty \quad \text{or} \quad G_3 = -G_o$$

The critical line value is given by $Z_o = Z_m$. The frequency dependent zeros are the solutions of Eq. (3.29).

$$0 = \epsilon^{j2\pi n} - \epsilon^{s\sqrt{\mu\epsilon}\, 2\ell} \qquad n = 0, \pm 1, \pm 2, \ldots \qquad (3.29)$$

$$\omega = \frac{\pi n}{\ell\sqrt{\mu\epsilon}}$$

However, this term does not present a serious problem. It is a term which is common to all first-order minors. This type of zero is allowed since it will

cancel when the ratio of the minors to the determinant are taken. Furthermore, if the system has a slight loss, i.e., if zeros on the jω axis are considered as physically nonrealizable as Bode does, the zeros are shifted into the left half s plane and cannot contribute to instability. Therefore, the ideal system is stable and does meet the criteria for realizability.

It should be pointed out that arguments based on the finite behavior of the input impedances are not sufficient to ensure stability. The condition is only a necessary condition. This may be shown by considering one of the termination zeros of Eq. (3.28). The input impedance does remain Z_1 and gives no indications of stability problems. However, if the input impedance of both lines is tested this difficulty is removed.

3.6 Stability Conditions for the Double-Pair Transformer with Tunnel Diode Terminations

It was stated earlier that it is impossible to obtain frequency independent reflectionless transmission if the terminations are complex for real frequencies. The stability analysis must involve the general solution for voltages and currents.

The stability analysis may be based on two different arguments. The first of these involves the coefficients directly and expresses the constraint due to their finiteness.

$$k_i(t) = \mathcal{L}^{-1}\left(\frac{V_o(s)}{G_o Z_s}\,\frac{\Delta_{1i}(s)}{\Delta(s)}\right) < \infty \qquad (3.30)$$

$$i = 1, 2, 3, 4$$

$$0 < t < \infty$$

If Z_s is positive real this condition is equivalent to a restriction of the location of those roots of $\Delta(s)$, which are not also roots of the minors of the first row.

$$\Delta(s) = 0 = \prod_{i=1}^{r} C(s - s_i) \tag{3.31}$$

(a) $\mathcal{R}e(s_i) \leq 0$

(b) if $\mathcal{R}e(s_i) = 0$ $\quad s_i \neq s_j$

In addition to these constraints frequency independent zeros must be avoided.

An alternative to this method is found in the driving point immittance criteria developed in the previous sections. Since the circuit involves two loops, both should be tested. This means that two sets of coefficients must be defined. The labor involved seems larger than that of the determinant criterion just mentioned, and the latter method is preferred for analytic examination. The roots of Eq. (3.32) must be investigated.

$$\Delta = Z_s Z_1 Z_2 Z_3 \Big[[(G_s + G_o)(G_3 + G_o) - G_m^2][(G_1 + G_o)(G_2 + G_o) - G_m^2]$$

$$(\epsilon^{sb} - 1) + [(G_s - G_o)(G_3 - G_o) - G_m^2][(G_1 - G_o)(G_2 - G_o) - G_m^2]$$

$$(\epsilon^{-sb} - 1) + 4(G_o^2 - G_m^2)(G_1 + G_s)(G_2 + G_3)\Big] \tag{3.32}$$

where

$$b = \sqrt{\mu\epsilon}\, 2\ell$$

The equation may be specialized by stipulating that the system is terminated for reflectionless transmission when all reactive termination components are removed, or, equivalently, if $\omega = 0$.

Cases II and III ($G_3 < 0$).

If it is assumed that the network is driven by a perfect voltage generator, ($Z_s = 0$), Eq. (3.32) reduces to:

$$0 = (G_3 + G_o)[(G_2 + G_o)(G_1 + G_o) - G_m^2](\epsilon^{sb} - 1)$$
$$+ (G_3 - G_o)[(G_2 - G_o)(G_1 - G_o) - G_m^2](\epsilon^{-sb} - 1) \qquad (3.33)$$
$$+ 4(G_o^2 - G_m^2)(G_2 + G_3)$$

By using Eq. (2.12) this expression may be reduced to the form given by Eq. (3.34).

$$0 = (G_3 + G_o)G_o(G_1 + G_2)(\epsilon^{sb} - 1) + 2(G_o^2 - G_m^2)sC_3 \qquad (3.34)$$

or

$$0 = G_o(G_o - G_2 + sC_3)(G_1 + G_2)(1 - \epsilon^{-sb}) + 2(G_o^2 - G_m^2)sC_3 \qquad (3.35)$$

The last term may be neglected if

$$G_o(G_1 + G_2) > 2(G_o^2 - G_m^2) \qquad (3.36)$$

This condition is satisfied if the lines are tightly coupled. Hence,

$$\Delta \approx G_o(G_o - G_2 + sC_3)(G_1 + G_2)(1 - \epsilon^{-sb}) = 0 \qquad (3.37)$$

Right half-plane zeros are avoided if:

$$G_o > G_2 \qquad (3.38)$$

The stability condition is identical to the condition derived in Example 3.2 for an uncoupled system. It indicates that region III is unstable. However, it

also allows the conclusion that Region II is stable for a tightly coupled system. Furthermore, the coupling results in a more favorable restriction on the line impedance.

$$Z_o < \frac{Z_2}{1 - \left(\frac{Z_m}{Z_o}\right)^2} \tag{3.39}$$

The frequency independent zeros are to be examined next. They cannot cause a restriction of an entire region, but can contribute singular points only. Furthermore, since they are frequency independent and since it was assumed that terminations were chosen in such a way that the reflectionless transmission conditions are satisfied for zero frequency, these zeros must be the same as those derived for the ideal resistance terminated case. An exact solution of the preceding equations shows this to be true.

Case I ($G_2 < 0$).

The arguments for this termination follow very closely along the same lines are those presented for the previous case. However, due to the occurrence of additional terms the analytic treatment becomes considerably more difficult. The presentation offered in this section substantiates the opinion that stable configurations exist for this case. The final answer should be based on computer evaluation of numerical descriptions.

$$\begin{aligned}\Delta = {} & (G_3 + G_o)[(G_2 + G_o)(G_1 + G_o) - G_m^2](\epsilon^{bs} - 1) \\ & - (G_3 - G_o)G_m KsC_2(\epsilon^{-bs} - 1) \\ & + 4sC_2(G_o^2 - G_m^2)\end{aligned} \tag{3.40}$$

The last term may be neglected for tightly coupled lines. This follows from a comparison of the first term in Eq. (3.40) with the last term in that equation. The approximated form of Eq. (3.40) becomes

$$\frac{-\Delta}{\epsilon^{-bs} - 1} = (G_3 + G_o)[(G_2 + G_o)(G_1 + G_o) - G_m^2]\epsilon^{bs} \qquad (3.41)$$

$$+ (G_3 - G_o)G_m KsC_2$$

However, in this region the approximation of Eq. (3.42) holds.

$$|G_m K| = G_1 - G_o \approx 0 \qquad (3.42)$$

This allows the suppression of the second term. The stability criterion may now be applied to Eq. (3.43).

$$\begin{aligned} T(s) &= (G_1 + G_o)(G_o - G_3 + sC_2) - G_m^2 \\ &= sC_2(G_1 + G_o) + (G_o - G_3)(G_1 + G_o) - G_m^2 \end{aligned} \qquad (3.43)$$

$$G_3 < \frac{G_o G_1 + G_o^2 - G_m^2}{G_o + G_1} \qquad (3.44)$$

This may be reduced somewhat more by eliminating G_1.

$$G_3 < \sqrt{G_o^2 - G_m^2}$$

or (3.45)

$$R_3 > \sqrt{Z_o^2 - Z_m^2}$$

Therefore, the conclusion is that some stable region exists for this case.

Case IV ($G_1 < 0$, $G_3 < 0$)

The two-tunnel diode case is again not solvable by analytic means unless approximations are made. The coefficient determinant is

$$\Delta = (sC_3 + \frac{G_m}{K})[(G_2 + G_o)(sC_1 + 2G_o - G_mK) - G_m^2](\epsilon^{sb} - 1)$$

$$- (sC_3 - G_2 - G_o)s \frac{C_1 G_m}{K} (\epsilon^{-sb} - 1) + 4sC_3(G_o^2 - G_m^2) \tag{3.46}$$

A very rough approximation may be obtained by considering a tightly coupled high gain case. The second and third terms become small for this situation. The stability condition is expressible as an upper bound on the voltage transfer function.

$$K < 2 \frac{G_o}{G_m} - \frac{G_m}{G_2 + G_o} \tag{3.47}$$

The condition may violate the assumption involved in the approximation. It cannot be used to prove stability. The question of stability for this case will be left unanswered here. However, it must be considered that two tunnel-diode structures have already been shown to be stable. This follows from case III. The sending generator for this case may be replaced by a negative resistor without difficulty. The arguments presented in the preceding sections allow the conclusion that stable networks can be found which utilize tunnel diodes as terminations. The allowable intercoupled structures will be restricted by stability conditions. However, stable systems remain which exhibit voltage and current gain.

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4. NEGATIVE RESISTANCE REALIZATION

The effects on system stability due to a capacitor in parallel with a constant negative resistance have been studied. The physical realization of a negative resistance, which is independent of current and frequency is questionable. The nonlinearity of the active element and its inaccessible reactances will cause deviations from the ideal model. A measure of these deviations and their effects is needed.

4.1 Constancy of the Negative Resistance

In order to investigate the negative resistance region of tunnel diodes special measurement techniques are necessary. The circuit shown in Fig. 4.1 was found helpful. Resistor R_4 is a microwave resistor contained in a high-frequency jig. The bridge initially is balanced without the device to be measured. Connection of the unknown after balancing yields the desired display.

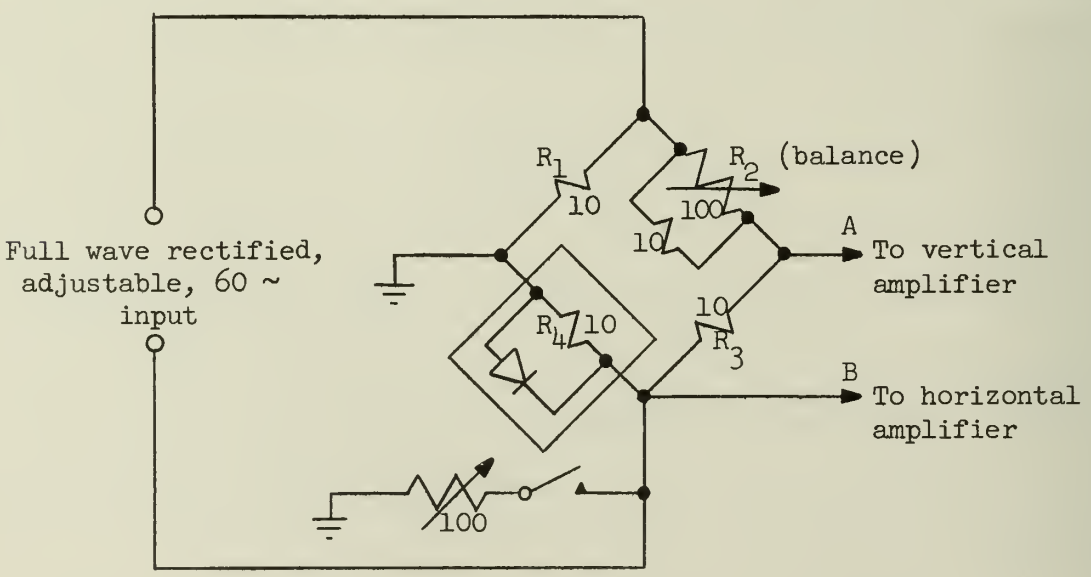

FIGURE 4.1. NEGATIVE RESISTANCE MEASUREMENT

Since the tunnel-diode characteristic is a continuous, well-behaved function a piecewise-linear approximation of this curve for a given voltage-current range is always possible. The accuracy of the approximation depends of course on the number of line segments used. The problem at hand is, however, somewhat different. The negative resistance region is to be approximated by a single line segment, and the voltage range for which this approximation is valid is to be determined. But the phrase, "an approximation is valid," implies validity within some specified error. Tolerances must be mentioned. Furthermore, since diode characteristics change with production processes and bulk materials a variety of devices should be examined. An exhaustive treatment is not intended here; however, enough material was collected to draw some conclusions. Figure 4.2 indicates data obtained from three diodes produced from different bulk materials. A reasonably linear region of 50 millivolts exists for the germanium diode. This increases to 80 millivolts for the silicon diode and 100 millivolts for the gallium arsenide unit. The increases are not as large as one would expect from the shifts in the peak voltages. Data on the variation of the negative resistance region for a given tunnel diode is represented by Fig. 4.3. The constancy of the negative slope in the region of interest is rather surprising. In order to emphasize this the diodes were measured individually. The results indicated a negative resistance of 35.0 $\pm$ .5 per cent at the inflection point. Furthermore, since considerable uniformity occurs from diode to diode, extension of the investigation to the Goto-pair seemed natural.

In order to obtain useful data it was decided to change to a somewhat slower five milliampere gallium arsenide diode (RCA TD176). A high-frequency jig was designed to hold the diodes during measurement. Results are indicated in Figs. 4.5 and 4.6.

FIGURE 4.2. VARIATIONS OF CHARACTERISTICS DUE TO BULK MATERIAL CHANGES

Legend: Traces read from left to right at peak current denote:

(a) 5-ma RCA germanium diode
(b) 5-ma GE silicon diode
(c) 5-ma RCA gallium arsenide diode

Horizontal scale: 50 mv/division
Vertical scale: Approximately 1.2 ma/division

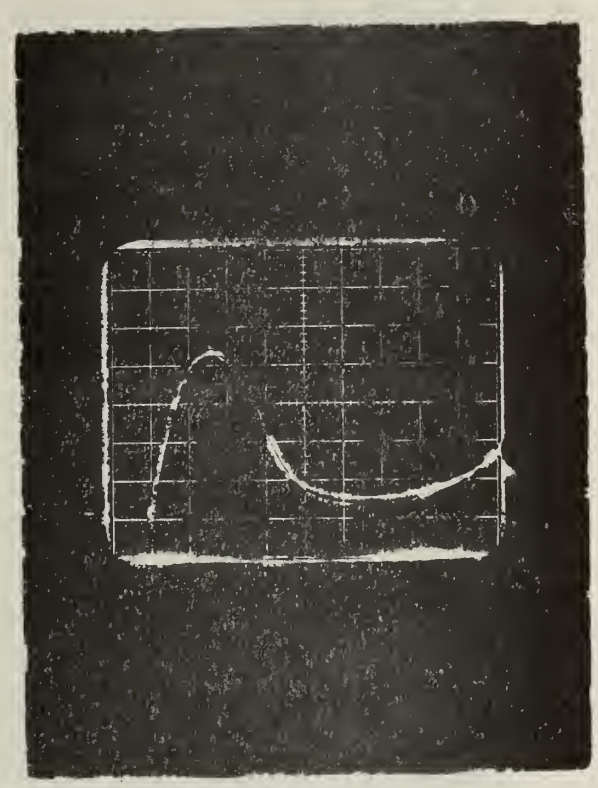

FIGURE 4.3. TOLERANCE BEHAVIOR OF GALLIUM ARSENIDE DIODES (5 SAMPLES) (5-ma Ga As DIODES, MS 1550 MICROSTATE ELECTRONICS CORPORATION)

Horizontal scale: 100 mv/division
Vertical scale: Approximately 1.2 ma/division

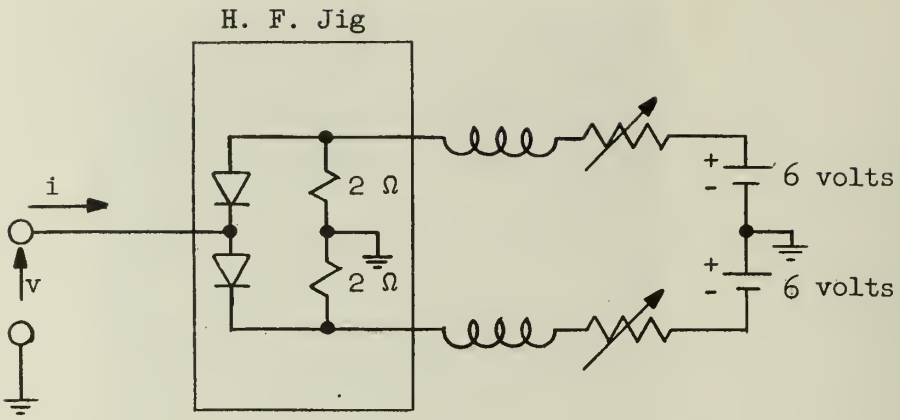

FIGURE 4.4. GOTO-PAIR CIRCUIT

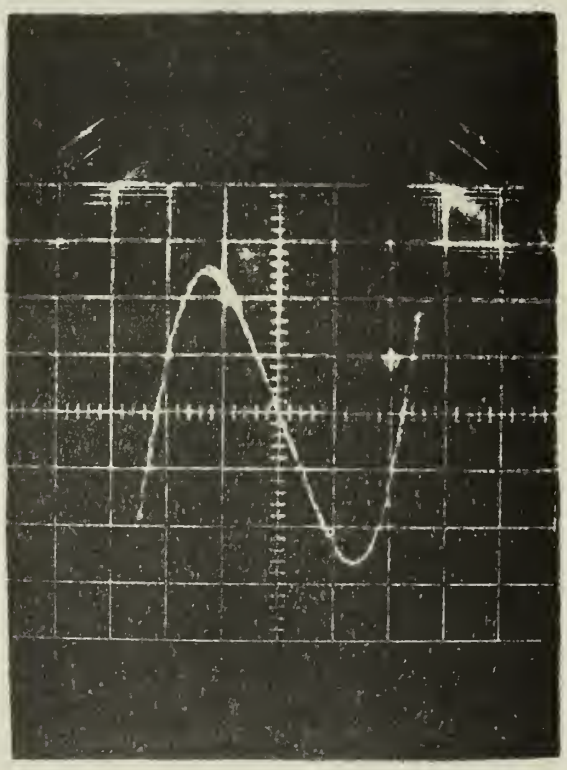

FIGURE 4.5. GOTO-PAIR CHARACTERISTIC. THE TWO TD176 DIODES WERE BIASED TO ± 280 MILLIVOLTS. ONE HORIZONTAL DIVISION IS THUS ROUGHLY 125 mv.

Vertical scale: Approximately 2 ma/division

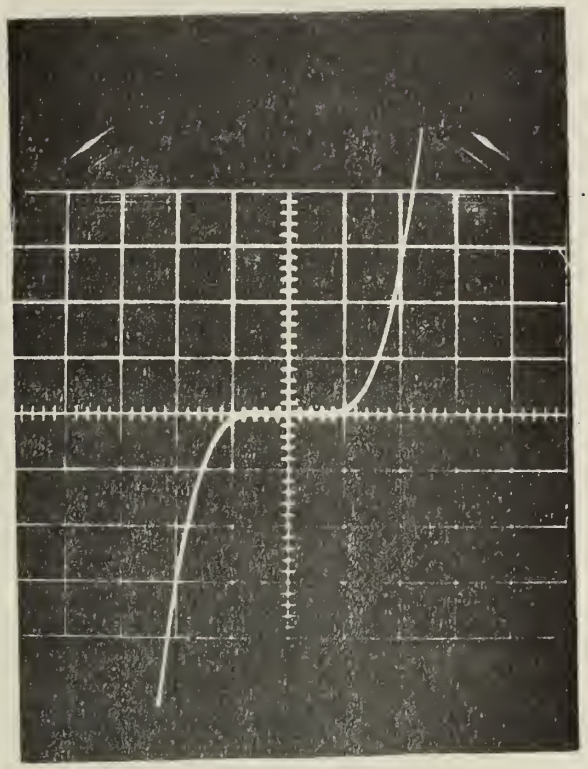

FIGURE 4.6. GOTO PAIR OF FIG. 4.5 IN PARALLEL WITH 30.4 OHMS

Scale: Same as Fig. 4.5

The data of Fig. 4.6 were obtained by shunting the input terminal of the Goto pair with a decade resistance and adjusting its resistance until an infinite incremental resistance was obtained in the negative region of Fig. 4.5.

The conclusions based on these experiments are:

(1) A constant negative incremental resistor can be obtained from suitable tunnel diodes.

(2) The slope in the negative resistance region of the diode shows much less tolerance sensitivity from unit to unit than expected.

(3) Goto-pair connection results in nearly perfect negative resistances for low frequencies.

4.2 Frequency Dependence of the Negative Resistance

The tunnel-diode equivalent circuit has been treated as a negative resistance in parallel with a capacitance. A more accurate circuit would incorporate the series resistance and inductance of the diode. The neglect of the series resistance is justified by a magnitude comparison to the negative resistance. However, a knowledge of its value is useful in the establishment of the figure of merit of the diode. The resistive cutoff frequency is given by:

$$f_{co} = \frac{1}{2\pi |-R| C} \sqrt{\frac{|-R|}{R_s} - 1} \qquad (4.1)$$

R_s - series resistance of the diode

The neglect of the diode reactance is at best a first-order approximation. The basic case inductance may be as low as 1×10^{-10} henries. If this type of diode is mounted properly, the inductive reactance will be about 1 ohm at 1500 megacycles. However, the positioning of the diode in the transmission line will result in other effects which are comparable to this one. Hence, if the inductance is to be included, the end effects must also be included. This type of analysis was not intended here. However, the value of the inductance serves to establish the self-oscillation frequency of the diode.

$$f_o = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{1}{((-R)C)^2}} \qquad (4.2)$$

The loading effect of the capacitance may be estimated. The frequency at which the resistance decreases to 90 per cent of its DC value is given by

$$f_1 = \frac{1}{6\pi} \frac{1}{RC} \qquad (4.3)$$

However, at this frequency the magnitude of the capacitive reactance is 1/3 R. Hence, the magnitude of the impedance of the RC-network is 95 per cent of its DC value. But the phase angle is about 20 degrees at f_1 and increasing. The inductances, which will exist in a real system, will tend to reduce the reactance. End effects may modify the situation. In any case, the frequency dependence of the negative resistor will appear as a phase distortion rather than a magnitude mismatch. An order of magnitude calculation indicates that for a 20-ohm resistor in parallel with 1 picofarad capacitance, phase shifts of less than 20 degrees will occur for frequencies below 2,700 megacycles.

4.3 Tolerance Effects

In the ideal model use was made of the fact that the coefficient determinant and its minors of the first row had the common factor $\epsilon^{s2b} - 1$. Therefore, the four coefficients, i.e., the ratio of the minors to the determinant, do not contain this term. The result is that the line will retain a flat frequency characteristic even at those frequencies which correspond to half-wave resonances of the line. If the system is mismatched the minors and the determinant no longer vanish. Their remainders are given by the following set of equations.

$$\Delta(n\,\frac{\lambda}{2}) = 4(g_3 + g_2)(g_s + g_1)(1 - g_m^2) \tag{4.4}$$

$$\Delta_{11}(n\,\frac{\lambda}{2}) = -2(g_3 + g_2)(g_1 + 1 - g_m^2) \tag{4.5}$$

$$\Delta_{12}(n\,\frac{\lambda}{2}) = 2(g_3 + g_2)(g_1 g_m) \tag{4.6}$$

$$\Delta_{13}(n\,\frac{\lambda}{2}) = 2(g_3 + g_2)(g_1 - 1 + g_m^2) \tag{4.7}$$

$$\Delta_{14}(n\,\frac{\lambda}{2}) = -2g_1g_m(g_3 + g_2) \tag{4.8}$$

where

$$n\,\frac{\lambda}{2} = \sqrt{\epsilon_r}\,\ell \qquad n = 0, 1, 2, \ldots$$

$$f(\lambda) = \frac{n}{2\sqrt{\mu\epsilon}\,\ell}$$

$$f(\lambda) = 1.5 \times 10^{10}\,\frac{n}{\sqrt{\epsilon_r}\,\ell}$$

The reflection coefficients of the two lines at this frequency are

$$\rho_1 = \frac{g_1 - (1 - g_m^2)}{g_1 + (1 - g_m^2)} \tag{4.9}$$

$$\rho_2 = -1 \tag{4.10}$$

Hence, in the neighborhood of the resonant frequency large disturbances must be expected. Minimization of their effects depends upon the type of application intended. If a flat amplifier is to be designed the resonant frequency should be selected at its maximum value; that is, the physical length of the line should be made short. The minimum length of line will depend on the end effects and, therefore, on the dimension of the line cross-section. The use of very thin dielectric sheets seems advantageous. This would allow a flat response between 10-2000 megacycles. An example is given in Fig. 4.7. If a pulse amplifier is to be designed the same arguments may be used. However, an alternative exists. Numerical evaluations indicate that the region of reflections is confined to

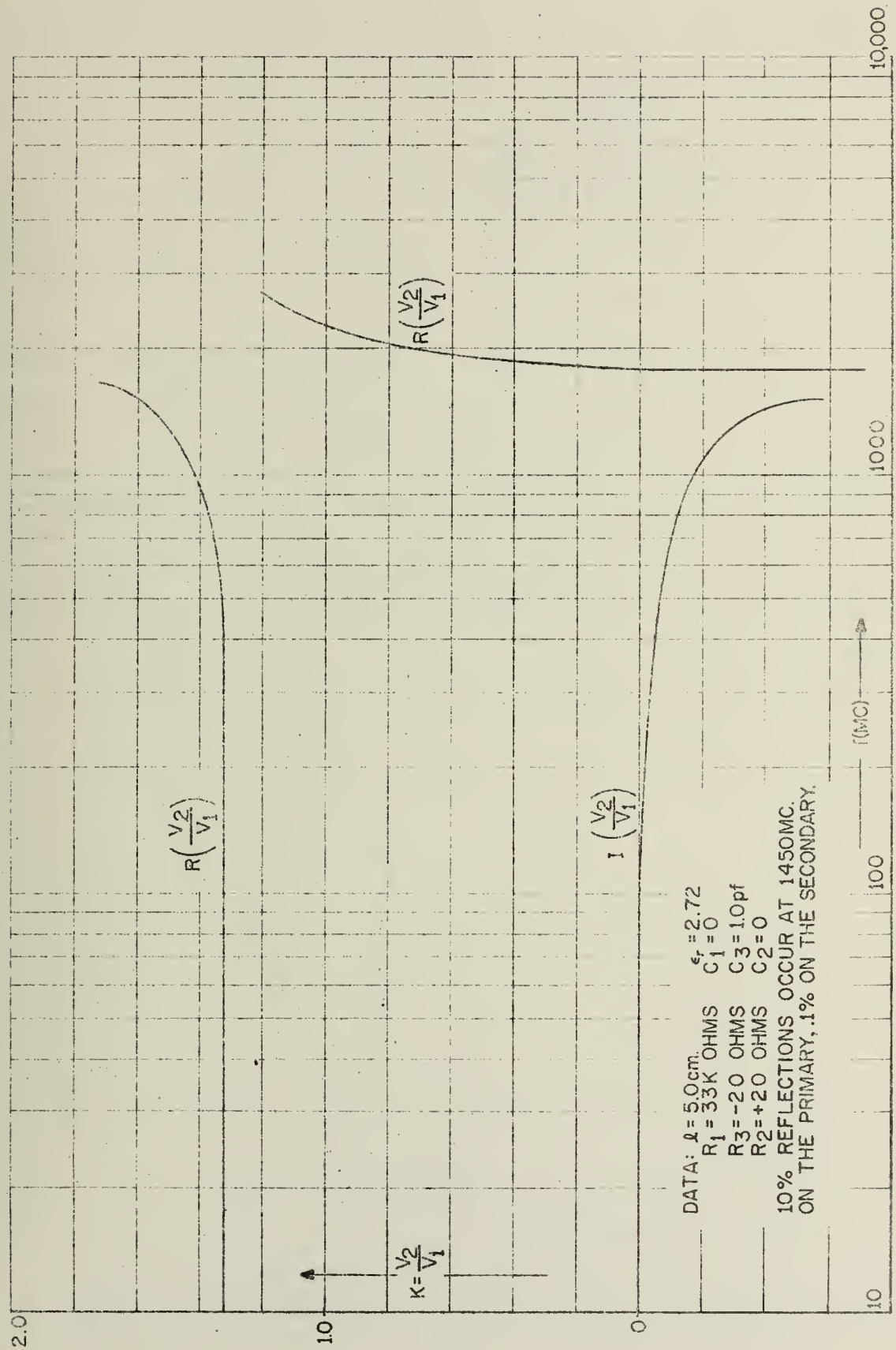

FIGURE 4-7: HIGH FREQUENCY PEAKING DUE TO SHORT TRANSMISSION LINE.

about 25 megacycles on both sides of the line resonance for a longer line. The frequency spectrum of the output pulse would therefore be distorted. However, this may not be too serious as far as the over-all system is concerned.

These difficulties also influence the low frequency response of the system. However, the data in Fig. 4.8 indicates that the disturbances may be confined to frequencies below ten megacycles.

From these arguments it may be concluded that the useful frequency range of the proposed system will be sharply influenced by the line length. The normal deterioration of system performance due to tolerance conditions will be small compared to the self-resonance effect. Flat pass-bands from 10-2000 megacycles seem to be designable. An example of parameter variations with respect to frequency is given in Figs. 4.9 through 4.13. The data are based on computer calculations.

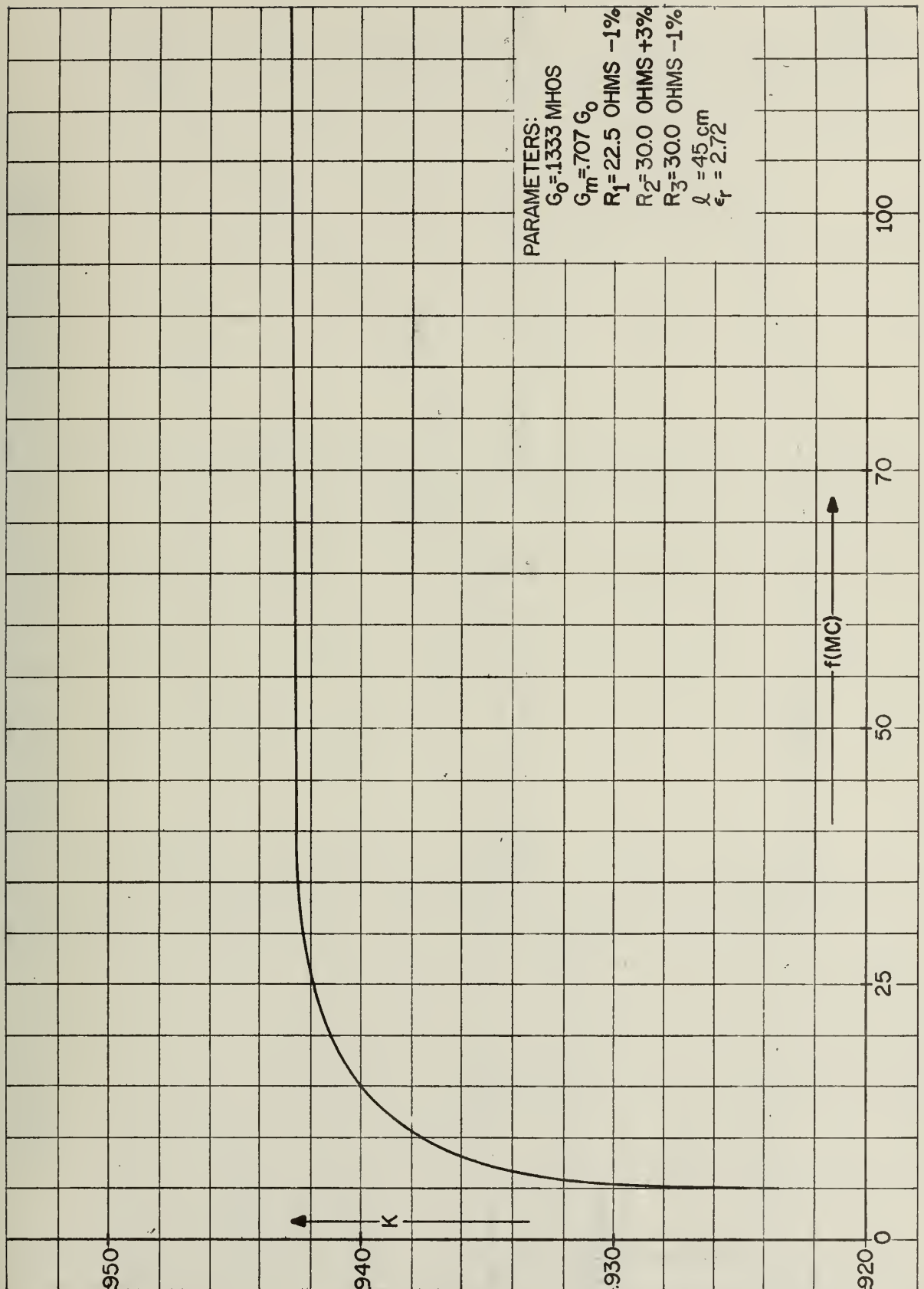

FIGURE 4-8: DC TOLERANCE EFFECTS. THE VOLTAGE TRANSFER OF THE IDEALLY TERMINATED SYSTEM IS K=.9427

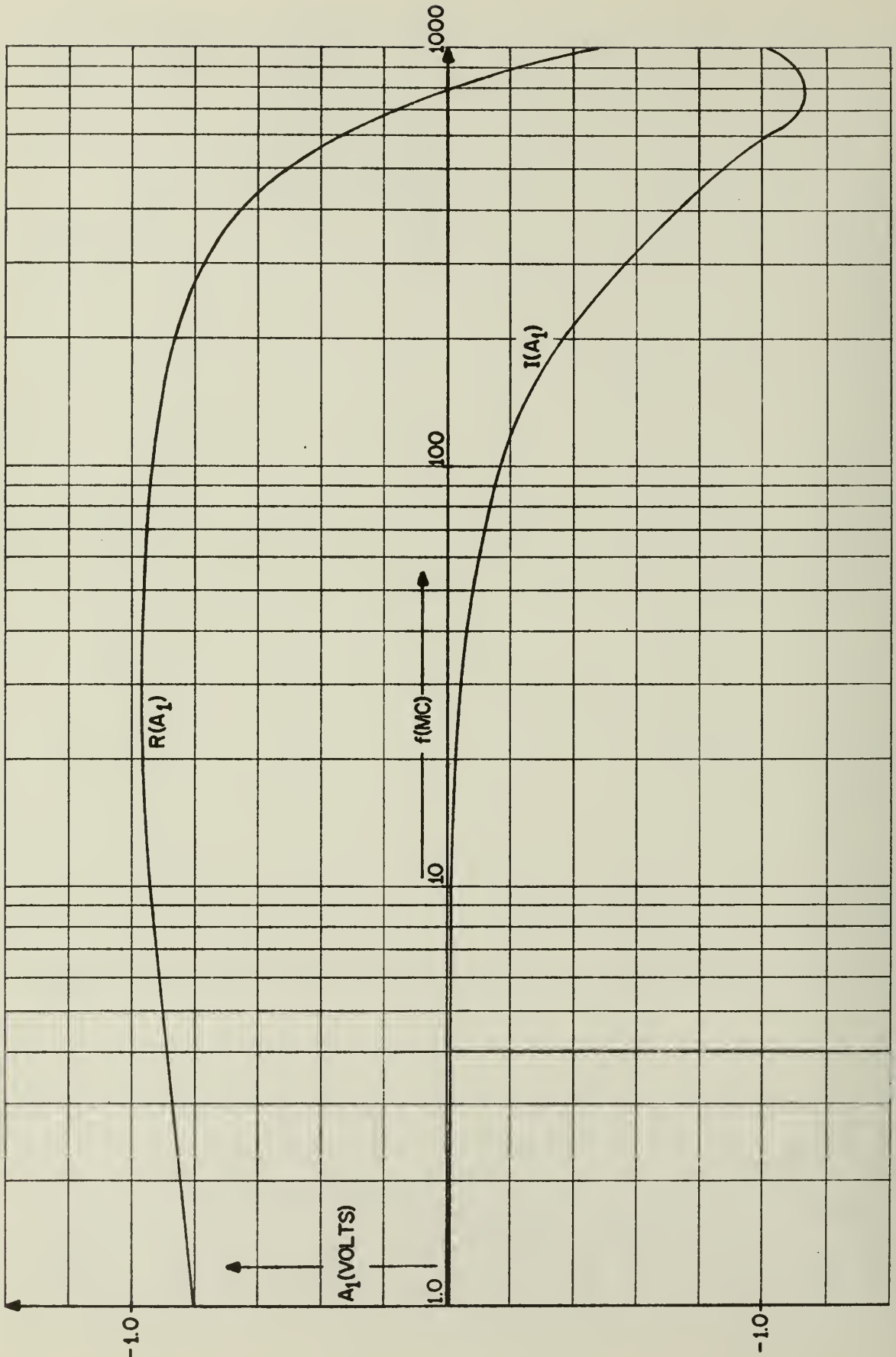

FIGURE 4-9: FORWARD AMPLITUDE OF PRIMARY LINE FOR UNIT EXITATION. THE BACKWARD AMPLITUDE IS LESS THAN $0.1|A_1|$ FOR THIS FREQUENCY RANGE. FOR DATA REFER TO FIGURE 4-11.

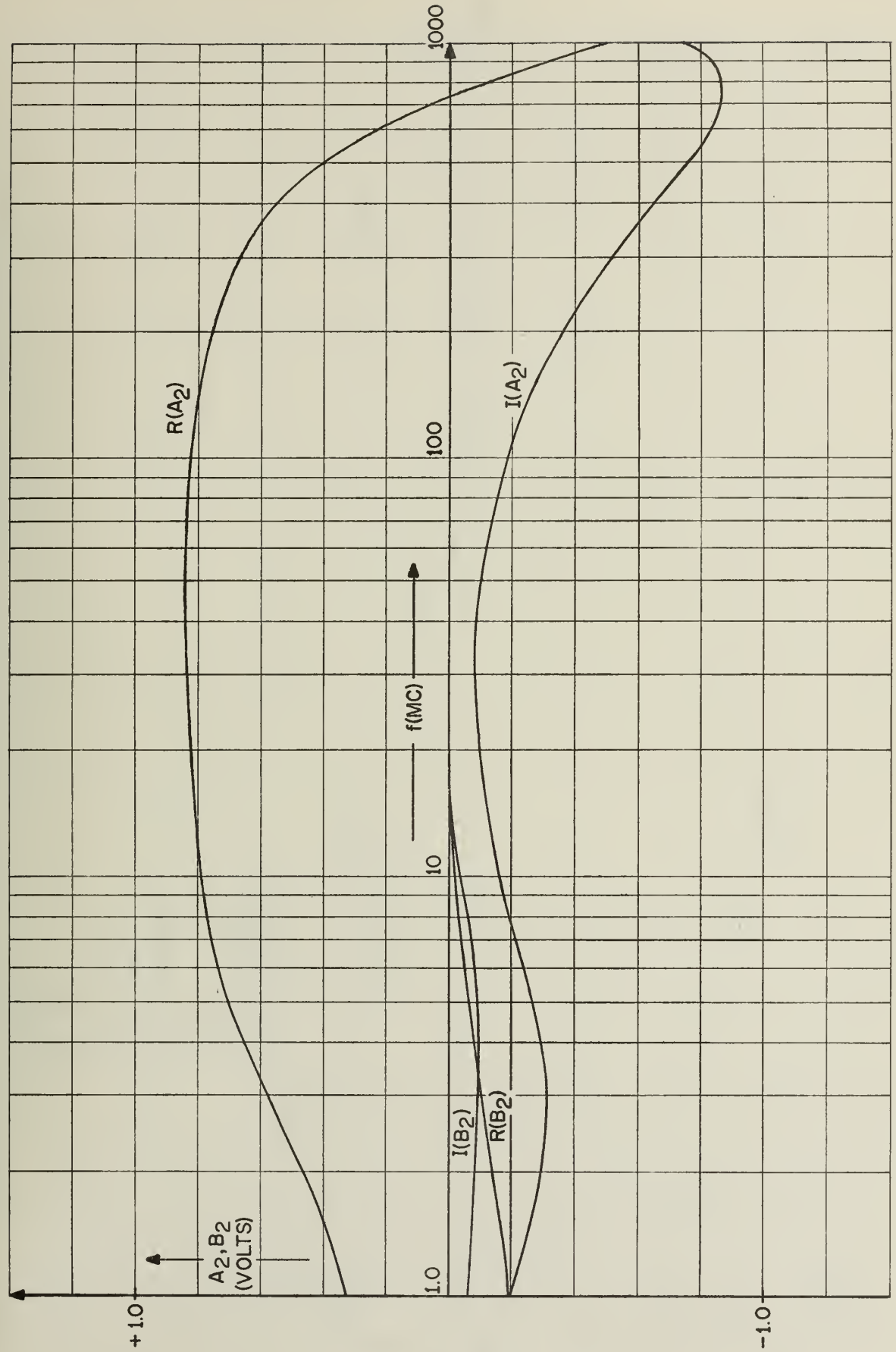

FIGURE 4-10 AMPLITUDES OF FORWARD AND BACKWARD WAVES ON SECONDARY LINE FOR UNIT EXCITATION. FOR DATA REFER TO FIGURE 4-11

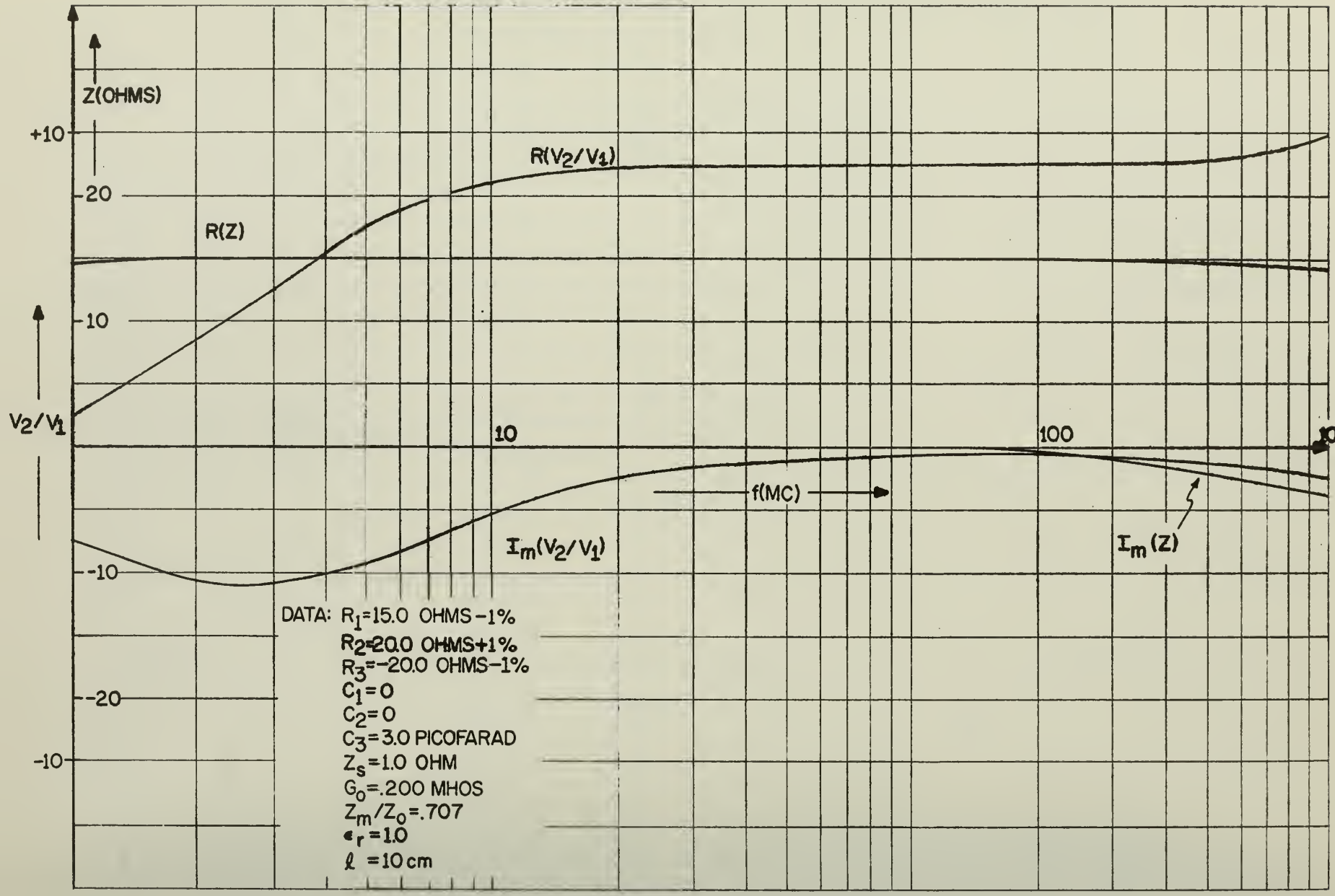

FIGURE 4-11: INPUT IMPEDANCE AND VOLTAGE TRANSFER. (REAL AND IMAGINARY PARTS)

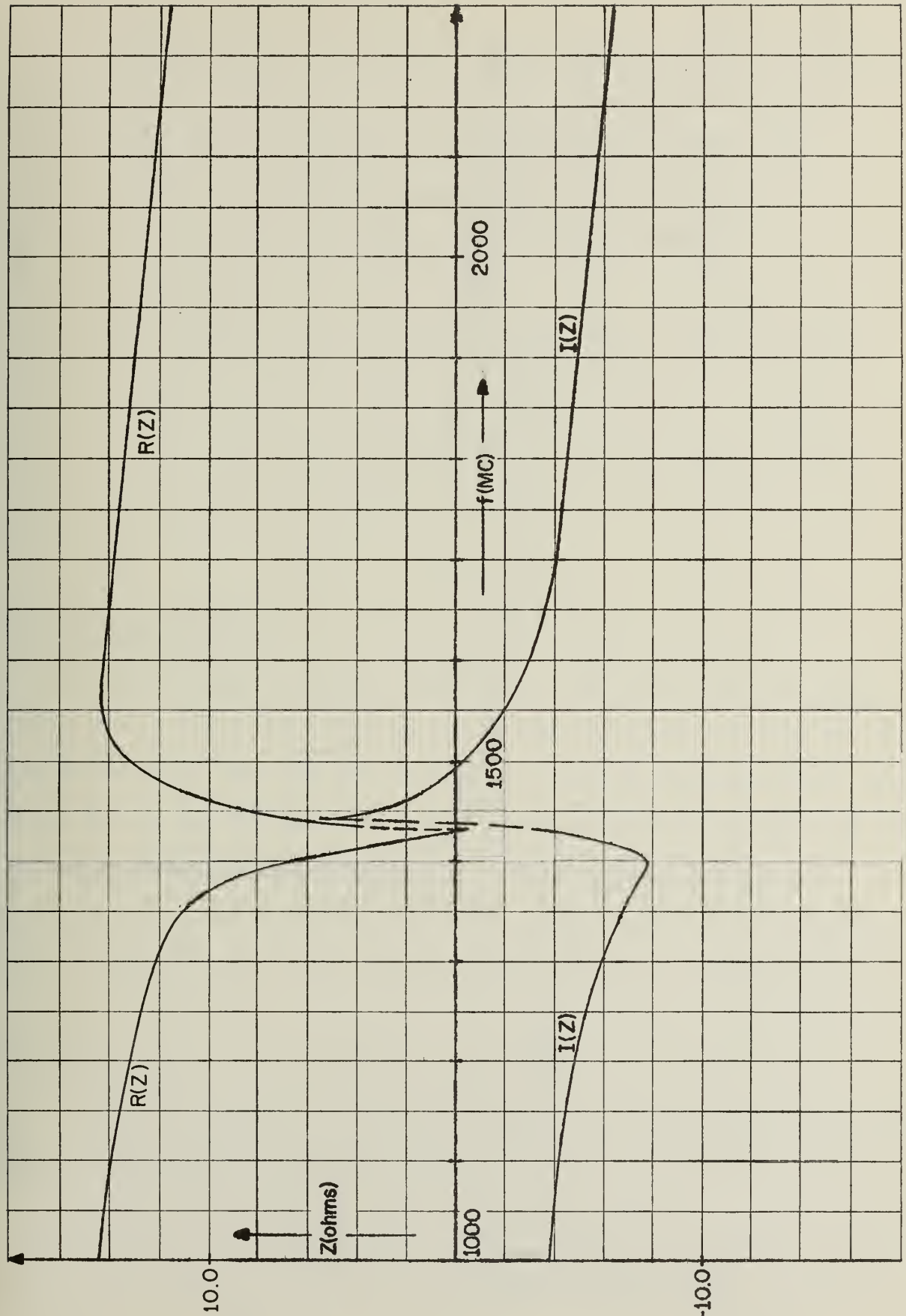

FIGURE 4-12: INPUT IMPEDANCE IN HALF-WAVE RESONANCE REGION. FOR DATA REFER TO FIGURE 4-11.

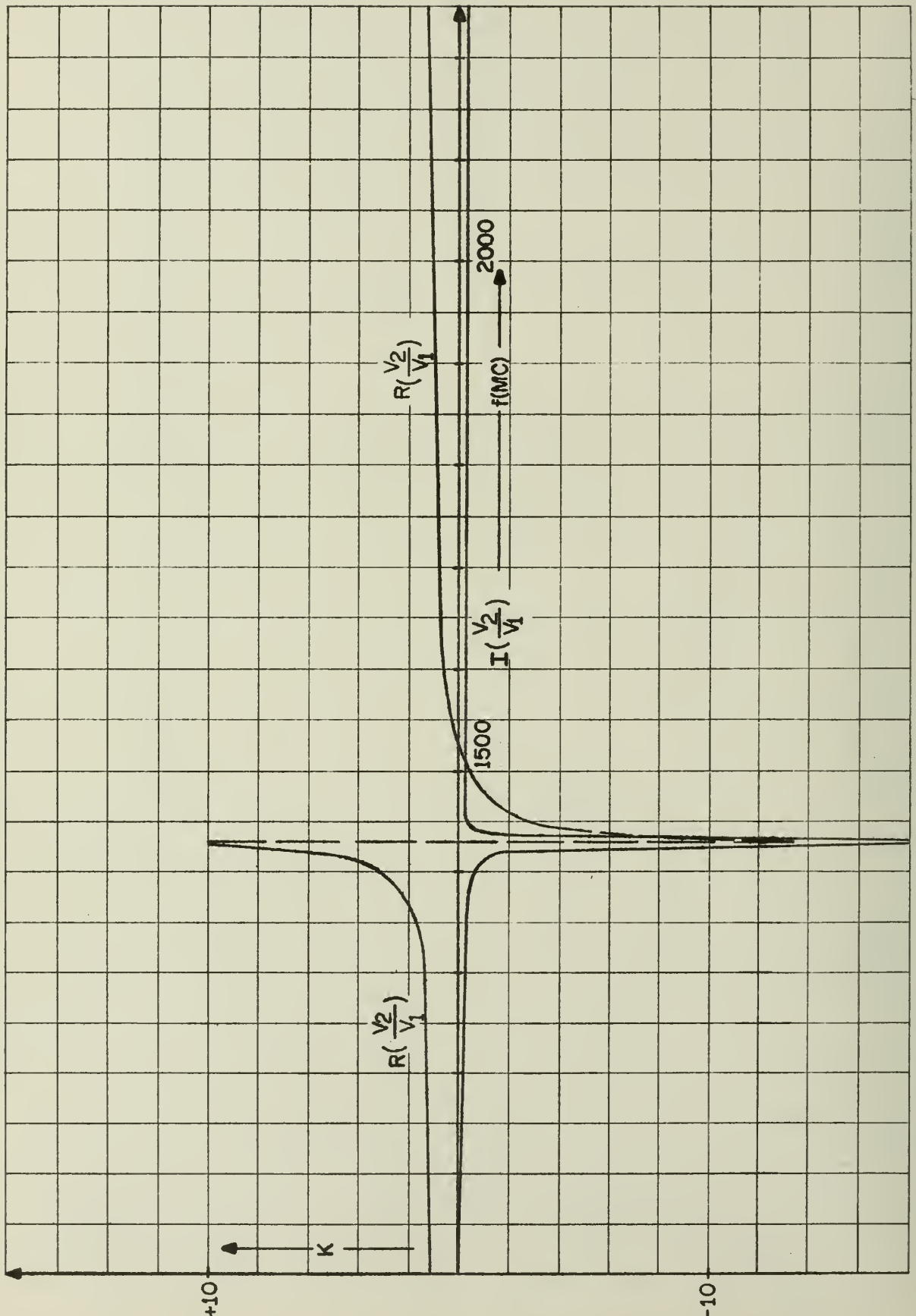

FIGURE 4-13: VOLTAGE TRANSFER IN HALF-WAVE RESONANCE REGION. FOR DATA REFER TO FIGURE 4-11.

5. TRANSMISSION LINE REALIZATION

The physical realization of the intercoupled transmission line is subject to two main criteria:

(1) A suitable dielectric can be found for the construction of intercoupled structures that will result in an over-all system that is lossless for all practical purposes.

(2) A geometry can be synthesized which enables the realization of low impedance lines with high coupling ratios. Production feasability, interconnectability, etc., are points which must be satisfied in order to achieve a useable system.

5.1 Dielectric Material

The choice of materials available for transmission line dielectric is fairly wide. Two of the better materials seem to be Rexolite 1422 and 2200. (Brand Rex Division, American Enka Corporation, Concord, Massachusetts). Dissipation factors for both materials are less than 0.001 for frequencies up to 10,000 megacycles. Rexolite 1422 has a dielectric constant of 2.55 and Rexolite 2200 a dielectric constant of 2.72 from 100 cycles to 10,000 megacycles. Standard photo-etching techniques may be used to process both materials.

5.2 T-Line

The characteristic and coupling impedance of the intercoupled symmetric line may be obtained by referring to Example 1.1.

$$Z_o \equiv \sqrt{\frac{L}{C}} = \frac{\overline{C} + \overline{C}_{12}}{\overline{C}(\overline{C} + 2\overline{C}_{12})} \sqrt{\mu\epsilon} \qquad (5.1)$$

$$Z_m \equiv \sqrt{\frac{L_{12}}{C_{12}}} = \frac{\bar{C}_{12}}{\bar{C}(\bar{C}+2\bar{C}_{12})}\sqrt{\mu\epsilon} \qquad (5.2)$$

In order to have a tightly coupled assembly, which is beneficial if high voltage and current transfers are desired, $\bar{C}_{12}$ should be made as large as possible. However, since a power amplifier should involve the transition from low peak current diodes to high peak current diodes, i.e., the terminations of the secondary line are low impedances, Z_o must be small and, therefore, $\bar{C}$ must also be large. A line which satisfies these conditions is indicated in Fig. 5.1.

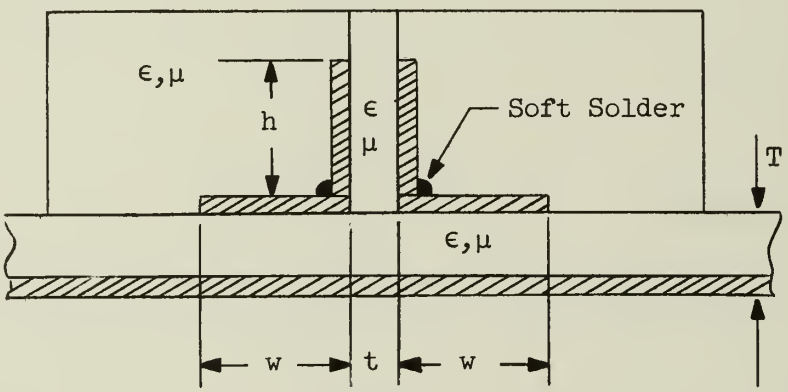

FIGURE 5.1. THE T-LINE

The line is assembled from two separately etched printed-circuit boards, and can be constructed to fairly close mechanical tolerances. The dielectric cover is provided to eliminate surface wave propagation due to the air-dielectric boundary.[5] The available range of Z_o is good and the ratio Z_m/Z_o may be made to approach unity. Disadvantages are the lack of self-shielding, the necessity for the cover and the unavailability of exact solutions for the capacitances involved. Approximate formulas for the impedances may be obtained if fringing is neglected.

$$Z_o \approx \sqrt{\frac{\mu}{\epsilon}} \frac{T}{w} \frac{1 + \frac{hT}{wt}}{1 + 2\frac{hT}{wt}} \tag{5.3}$$

$$Z_m \approx \sqrt{\frac{\mu}{\epsilon}} \frac{T}{w} \frac{\frac{hT}{wt}}{1 + 2\frac{hT}{wt}} \tag{5.4}$$

Orders of magnitude involved may be obtained from the following example.

Example 5.1.

The coupled line is to be designed in such a way that $Z_o\sqrt{1 - \left(\frac{Z_m}{Z_o}\right)^2}$ is equal to 22 ohms. This corresponds to the negative resistance of a 5-milliampere germanium tunnel diode. The coupling is to be such that $2Z_m = Z_o$. Then:

$$Z_o = \frac{2}{3} \frac{377}{\sqrt{\epsilon}} \frac{T}{w}$$

$$\frac{T}{w} = \frac{22\sqrt{3\epsilon}}{377}$$

If Rexolite 2200 is used this yields

$$w \approx 6T$$

or

$$T = 1/16", \; w = 3/8"$$

These numbers are reasonable for fabrication purposes.

5.3 Broadside-Coupled Strip Transmission Line

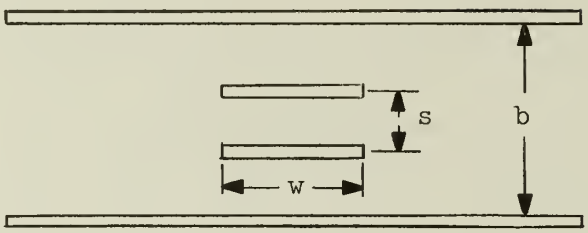

FIGURE 5.2. BROADSIDE-COUPLED TRANSMISSION LINE

The coupled system shown in Fig. (5.2) has been described in the literature. The following approximate solutions are given if $w/s \geq .35$:

$$Z_{oe} = \frac{188.3}{\sqrt{\epsilon_r}} \frac{K(k')}{K(k)} \tag{5.5}$$

$$Z_{oo} = \frac{296.1}{\frac{b}{s}\sqrt{\epsilon_r}\tanh^{-1} k} \tag{5.6}$$

$$\frac{w}{b} = \frac{2}{\pi}\left[\tanh^{-1}\sqrt{\frac{k\frac{b}{s} - 1}{\frac{1}{k}\frac{b}{s} - 1}} - \frac{s}{b}\tanh^{-1}\left(\frac{1}{k}\sqrt{\frac{k\frac{b}{s} - 1}{\frac{1}{k}\frac{b}{s} - 1}}\right)\right] \tag{5.7}$$

where

$K(k)$ is a complete elliptic integral of the first kind

and

$$k' = \sqrt{1 - k^2}$$

Z_{oe} is the impedance from one line to ground if both lines are excited from the same source. Z_{oo} is the impedance from one line to ground if one line is excited

by a voltage +V and the other by a voltage -V. The suggested error due to a single approximation should not exceed one per cent and should be considerably more accurate for the geometries of interest here.

The relationship between the even and odd mode impedances to the intrinsic impedance Z_o and the coupling impedance Z_m used up to this point may be established by considering the circuit of Fig. 5.3.

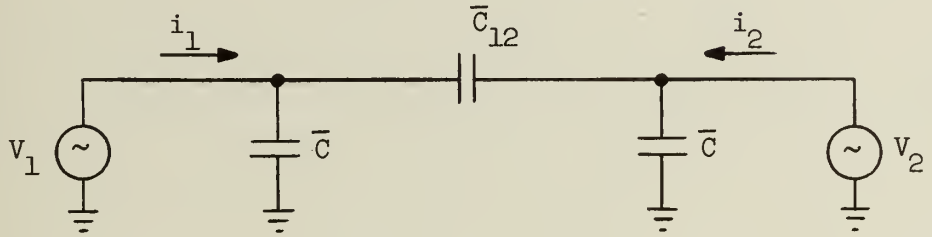

FIGURE 5.3. ODD AND EVEN MODE CONVERSION

For the even excitation the relationship is:

$$\frac{v_{1e}}{i_{1e}} = Z_{11} + Z_{12} = \frac{1}{s\bar{C}} \tag{5.8}$$

The corresponding relationship for the odd case is given by:

$$\frac{v_{1o}}{i_{1o}} = Z_{11} - Z_{12} = \frac{1}{s(\bar{C} + 2\bar{C}_{12})} \tag{5.9}$$

This yields:

$$Z_o = \frac{1}{2}(Z_{oe} + Z_{oo}) \tag{5.10}$$

$$Z_m = \frac{1}{2}(Z_{oe} - Z_{oo}) \tag{5.11}$$

It is in principal possible to select a value of Z_o and Z_m, determine from these data Z_{oe} and use tables to obtain k from the ratio $\frac{K(k')}{K(k)}$. If k is known $\frac{b}{s}$ may be computed from Eq. (5.6). This allows the ratio w/b to be calculated from Eq. (5.7). There are several difficulties with this method. A low value of Z_o is desired. This means that k attains values which are very close to unity, which in turn demands the use of five or six place tables, which are not available. Furthermore, the above method presupposes that all values of b/s are satisfactory. This involves some practical difficulties. The use of printed-circuit boards of thickness t and 2t limits the allowed values to 3 and 5. The usefulness of other values of b/s is subject to the availability of different board sizes.

The above difficulties were resolved by selecting a value of b/s = 5 and calculating Z_o and Z_m as functions of k by means of a special computer program. The coupling ratio Z_m/Z_o for this type of calculation can no longer be specified independently. The results are indicated in Fig. 5.4. The line is useful for impedances between 40 and 20 ohms. For this range the coupling is larger than .500. Advantages over the T-line may be found in the self-shielding character, easier construction and highly accurate parameter designability.

5.4 Interconnection

Either one of the previously considered intercoupled structures seems to be useable for amplifier use. However, if subnanosecond responses are expected, the problem of signal transfer from block to block must be solved in order to obtain an overall system which is capable of operating at this speed. The standard interconnection by the use of flexible coaxial cable is somewhat awkward. The use of the cable implies the transition from flat line to coaxial

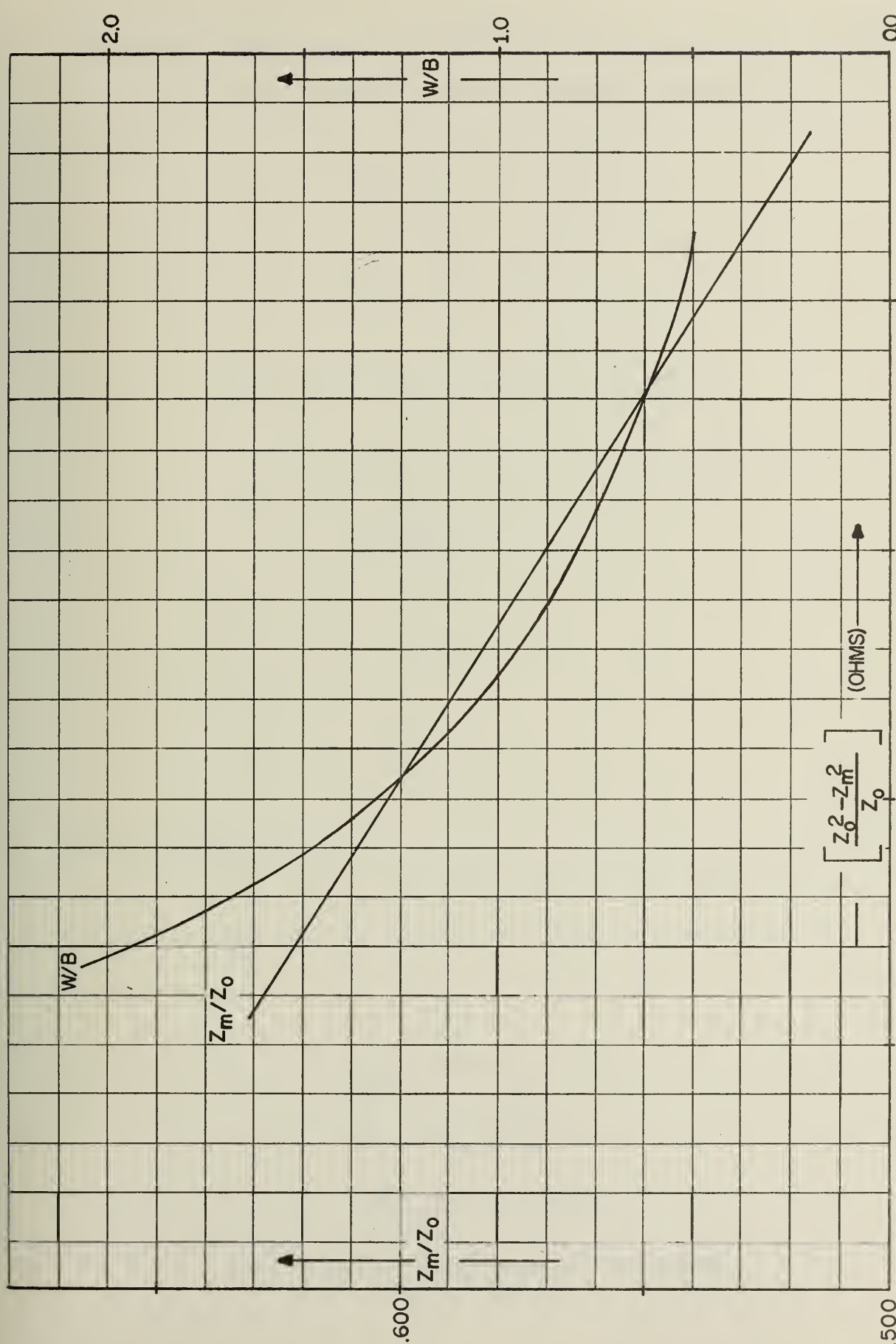

FIGURE 5-4: CHARACTERISTICS OF BROADSIDE COUPLED STRIP TRANSMISSION LINE FOR B/S=5, ϵ_r=2.72

line and will result in mismatch for some of the frequencies in the pulse spectrum. An alternate is offered by the use of tri-plate line.

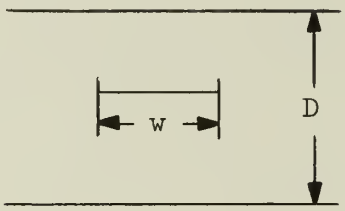

FIGURE 5.5. TRI-PLATE TRANSMISSION LINE

This line is characterized by the following formulas:

$$Z_o = 30\pi \frac{1}{\sqrt{\epsilon_r}} \frac{K(k')}{K(k)} \qquad (5.12)$$

$$k = \operatorname{sech} \frac{\pi w}{2D}$$

$$k = \tanh \frac{\pi w}{2D}$$

Its use decreases the mismatch due to interconnections and allows more flexibility since the value of Z_o is easily adjusted.

If the broadside-coupled line is used, the line shown in Fig. 5.6 is useful.

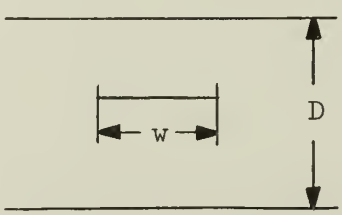

FIGURE 5.6. NONSYMMETRICAL TRI-PLATE LINE

By orienting two of these lines at right angles to the intercoupled structures suitable signal take-offs can be designed which introduce very little disturbance.

6. CONCLUSIONS

The arguments presented so far result in the conclusions that special lossy or lossless intercoupled transmission lines, when terminated by a combination of tunnel diodes and resistors may yield extremely wide-band amplifiers and impedance converters. Their physical realizability has been proven. The feasibility of constructing this type of assembly by the use of real components has been studied and has been found possible. Data from several experiments are given below as experimental verification of the theory.

Experiment 1:

This experiment was performed to test the behavior in region II. A single gallium arsenide diode was used to terminate the sending end of the secondary line. The line used was a T-line. Calculation of its parameters was based on the assumption of perfect field configurations. The magnitude of the negative resistance and a desired coupling ratio of $Z_m/Z_o = \frac{1}{\sqrt{2}}$ were taken as fixed design parameters. The results are indicated in Fig. 6.1.

The theoretical voltage transfer was calculated to be 1.13. The measured transfer is .92 so that an error of about 8 per cent is involved. This is reasonable. The power gain of this circuit is slightly larger than 1.7. The rise and fall times are diode limited.

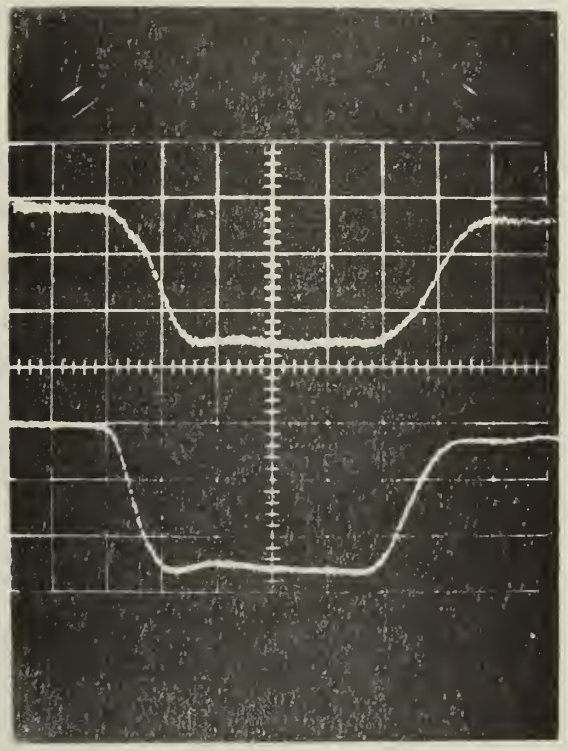

FIGURE 6.1. T-LINE (REGION II)

R_1 = 64.5 ohms

R_2 = 33 ohms

R_3 = -34.5 ohms

C_3 = 15.5 picofarad } TD176

R_s = 50.0 ohms

Z_o = 25.7 ohms w = .162"

Z_m = 18.2 ohms h = .390"

Rexolite 2200 (t = 1/32")

ℓ = 24"

Upper trace: Secondary line, receiving end
Lower trace: Primary line, receiving end

Scales: Horizontal: 1 nanosecond/division
Vertical: 50 mv/division

Experiment 2:

A T-line terminated by a germanium diode was used to test region I and high frequency response. A 24-inch line was used which was designed so that $\frac{1}{G_o}$ = 22 ohms. The coupling ratio was again taken as $1/\sqrt{2}$. The magnitude of the negative resistance was 86 ohms. A theoretical voltage transfer of .56 was calculated. The experimentally-determined transfer is found to be .50. The rise and fall times are very close to the response time of the Tektronix 661 sampling system (.35 nanoseconds).

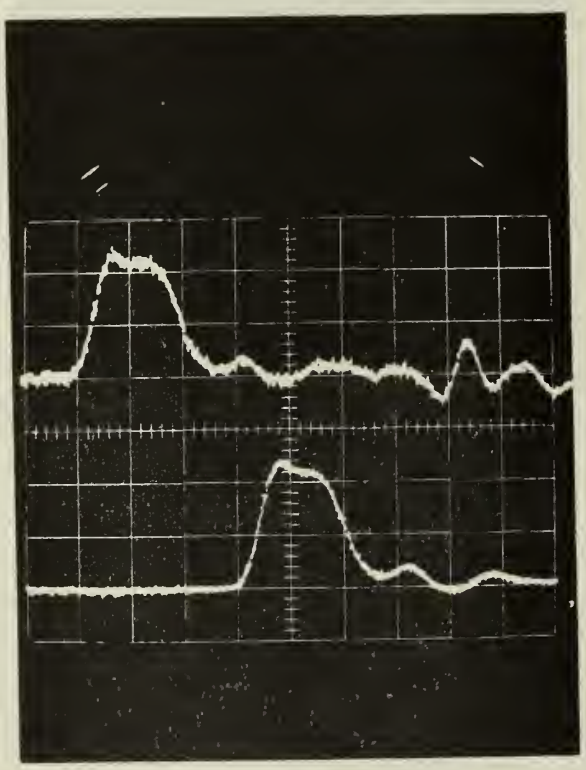

FIGURE 6.2 T-LINE (REGION I)

A 1.6 milliampere germanium diode ($|-R|$ = 86 ohms, C = 6.7 pf) was used as the termination of the receiving end of the secondary line.

Upper trace: Receiving end of secondary line.
Vertical scale: 10 mv/division

Lower trace: Receiving end of primary line
Vertical scale: 20 mv/division

The horizontal scale is 1 nanosecond/division for both traces.

Experiment 3:

In order to obtain a higher degree of control over the line parameters the broadside-coupled line was used. In this experiment the primary line is open circuited. Parameter sensitivity is thus unusually high. The negative termination was a Goto pair formed from two gallium arsenide units (TD176).

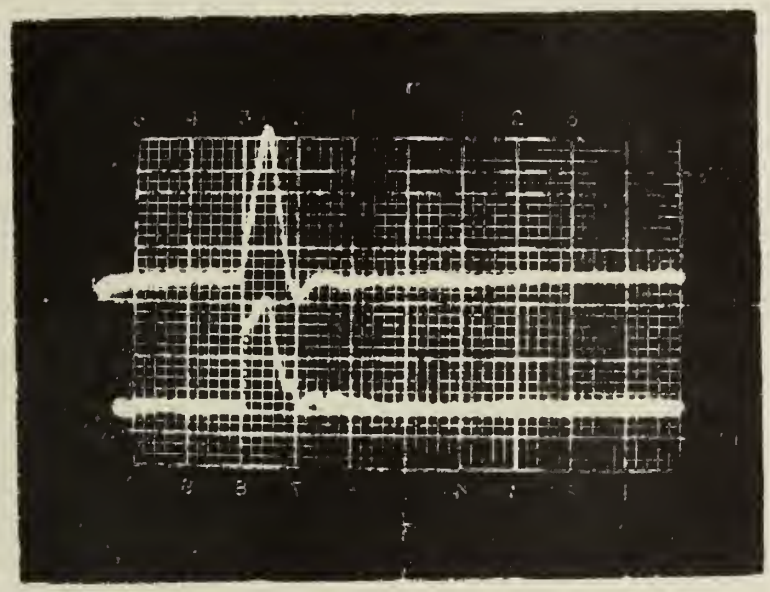

FIGURE 6.3. BROADSIDE-COUPLED LINE (REGION II)

R_3 = -30 ohms

C_3 = 30 picofarad

R_2 = 30 ohms

R_1 = ∞

R_s = 50 ohms

A line constructed from two 1/16" and one 1/32" pieces of Rexolite was used. The width of the central conductor was .216", the line length 18".

Upper trace: Output of secondary line.
Lower trace: Output of primary line.

Scale: Horizontal: 10 nanosecond/division
Vertical: 50 mv/division

A theoretical voltage transfer of 1.7 was calculated. The measured transfer is slightly larger than 1.5. The reflections on the primary trace are due to disturbances generated by scope loading. The results of the experiment substantiate the argument that voltage and current gain can occur simultaneously.

Attempted experiments in region III (inverting voltage transfer) were unsuccessful due to oscillations. However, these results had been predicted. Hence, the stability theory may be considered verified. Since all experiments were conducted with long sections of line and resulted in reasonable output pulse shapes, their use is permissible for pulse networks.

The primary goal of this investigation has been the synthesis of a wide-band tunnel-diode amplifier. This has been achieved. However, even though the synthesis of useful hardware is desirable, it is perhaps as important to recognize the introduction of distributed circuit concepts into the digital computer field. This, of course, is not original. However, inspection of attempted approaches described in the literature indicates that distributed circuit ideas have been introduced after lumped circuit treatments failed, i.e., they are used as extensions of conventional analysis or synthesis methods. This method does in general eliminate those networks which yield some desired characteristic because they employ distributed circuits and results in a class of circuits which performs satisfactorily even though it makes use of distributed circuits. An example of this extension is found in the following arguments. The amplifiers discussed earlier would yield, when suitably interconnected, the required system directivity mentioned in the introduction. During the experimental investigations it was found that if the negative termination is shifted to a positive one, by altering the bias for instance, transmission from the input port of the primary line to the output port of the secondary line ceases. Therefore, diode action can be simulated in this fashion. This type of response would

certainly enhance the desired unidirectional information flow. Investigations of this phenomena and its relationship to backward couplers are in progress. Many other examples may be found. Among them are the very important tunnel-diode-to-transistor-interconnection problem as well as gated high-frequency oscillators. Further extensions of the basic ideas presented can be expected.

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